The role of precious metals in portfolio diversification during the Covid19 pandemic: A wavelet-based quantile approach

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A B S T R A C T
In this study the relation between stock markets and precious metals during first wave of Covid-19 pandemic are investigated. We use a wavelet-based quantile procedure to investigate correlation between major stock markets of emerging countries (BRIC) and the United States. Our procedure reveals that precious metals offer market diversification opportunities during the period under consideration. In particular, it is found the gold, silver, platinum, and palladium, all serve as safe-haven assets during periods of market distress across short, medium and long investment horizons.

1. Introduction

In financial literature, the correlation across market returns plays a crucial role in portfolio diversification. Markowitz (1959) suggests that investors should assemble an asset portfolio that maximises expected returns for a given level of risk. According to the Markowitz model, risk-averse investors seek to minimise idiosyncratic risk by holding assets that are not perfectly and positively correlated, (for a recent review, see for example, Koumou, 2020). Since the seminal paper by Markowitz (1959), diversification as a portfolio strategy has become one of the major components of investment decision-making under uncertainty. Portfolio diversification strategies are crucial to dampen losses that investors may face due to price fluctuations in periods of market distress (see for example, Levy, 1978; Levy and Roll, 2010).

This paper adds to the growing literature on portfolio diversification in periods of financial distress. We are particularly interested in investigating the role of precious metals in portfolio diversification during the early period of the COVID-19 pandemic. In the literature a large number of empirical studies have shown that investors seek to increase the proportion of low-risk assets during a period of market turmoil (see for example, Joy, 2011; Daskalaki and Skiadopoulos, 2011; Arouri et al., 2015). The abundance of empirical evidence has prompted economists to seek explanations for the observed behaviour. For example, according to the “flight to safety” theory, during periods of financial turmoil, risk-averse investment managers seek to sell assets perceived as risky to purchase safer assets. In the literature, theoretical models that generate the flight to safety behaviour have been proposed. For example, according to the theoretical model used in a study by Vayanos (2004), risk-averse investment managers require higher risk premiums during periods of financial turmoil, which in turn drives down risky asset prices. This, ultimately, induces investors to sell assets that are perceived as risky and purchase safe ones instead. As per Caballero and Krishnamurthy (2008), increased uncertainty during a period of financial turmoil leads agents to sell risky financial claims in favour of safe claims. Brunnermeier and Pedersen (2009) provide a model in which traders become reluctant to take “capital intensive” positions in high-margin securities when funding liquidity is tight. The theoretical model explains the flight to quality behaviour being triggered by a sharp drop in liquidity provision for the high margin and more volatile assets. In the literature, representative agent models have also been used to explain the flight to quality behaviour. In these models, a “flight to safety” is typically defined as the joint occurrence of exogenous shocks leading to economic uncertainty with lower stock prices (induced by a cash flow or risk premium effect) and low real rates (through a precautionary savings effect) (Baer et al. 2020) (see also Beber et al., 2009; Bekker et al., 2009).

Against this background, the uncertainty brought about by the global spread of COVID-19 has heightened the market risk aversion in ways

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that have not been witnessed since the global financial crisis (OECD report, 2020). In this respect, a growing body of literature has found that the COVID-19 pandemic had an important effect on financial markets. For example, Baker et al. (2020) argue that in the U.S. stock markets’ volatility levels in the first quarter of 2020 surpassed those as last seen in October 1987 and December 2008 and, before that, in late 1929 and the early 1930s (see also Al-Awadhi et al., 2020; Alfaro et al., 2020; Zhang et al., 2020). For centuries, precious metals such as gold and silver have been perceived as safe investment assets during the period of bearish financial markets. However, the question has gained momentum during the COVID-19 pandemic since other types of low-risk assets, such as sovereign bonds for example that have been traditionally used to balance portfolios in periods of financial turmoil, have been traded with negative rates. At the same time, central banks have implemented expansive monetary policy measures by keeping low-interest rates to support the economy. The low-interest-rate environment has reduced the opportunity cost of precious metals with respect to other forms of investments. In addition, fiscal support to the economy has raised concerns about a long-term run-up of inflation caused by a widespread surge in government debt.

Against this background, the questions we ask in this paper are: Do precious metals provide valuable diversifying opportunities for equity portfolios during periods of financial turmoil? Consistent with modern portfolio theory, the issue involves answering the question: Are the returns of precious metals negatively correlated to stock returns? Also, how does this correlation change in relation to the stock market shocks? From the operational point of view, the question translates to the following: Does the sensitivity of precious metals to variation of stock market returns changes over the lower, upper, median quantiles of the precious metal distribution?

Finally, it is well known that different types of investors have different time horizons in their investing strategies. For example, day traders that seek to minimise the risk of their portfolios may have very different time horizons from pension fund managers that must typically ensure that eligible retirees receive the benefit they were promised. On the other side, hedge fund managers tend to invest in assets that can provide them good returns on investments within a short time. For this reason, they prefer liquid assets that allow them to shift portfolio allocation quickly. In this respect, evaluating the precious metal properties as potential portfolio stabilizers requires investigating a broad spectrum of different time horizons to reflect the investors’ heterogeneity. Accordingly, an additional issue we tackle in this paper is: How does the correlation between precious metals and stock markets changes over time and across investment horizons?

This paper considers stocks issued by four leading emerging stock markets of the BRIC countries (Brazil, Russia, India, and China) and the United States. The United States is considered in the paper since negative shocks to the US economy can potentially harm the global financial stability. On the other side, the group of BRIC are of interest because they are among the top 10 countries with the largest gold reserves in the world. China and India together account for around 40% of the total world gold bar and coin demand (World Gold Council, 2020). Russia has emerged as a major gold mining nation, and its central bank has built very substantial gold reserves over time.1

To address the issues above we proceed in two stages. In the first step we consider four precious metals, (namely, gold, silver, platinum, and palladium) and analyse if these commodities have offered portfolio diversification opportunities during the first wave of the COVID-19 outbreak in 2020 and how diversification properties (if any) changed across investment horizons. With this target in mind, we propose a novel methodology that combines the benefits of wavelet series expansions with the quantile estimation. We name it “wavelet quantile correlation” procedure, in short, “WQCOR”. The procedure can easily be carried out in two steps. The first stage involves using wavelet analysis to decompose the series of precious metals and stock market returns into components associated with different scale resolution. In the second step, the decomposed series are used as input variables to estimate the conditional quantile dependence between the precious metals and the stock markets under consideration. We are particularly interested in estimating how the conditional correlations change over the quantiles of the precious metal distributions in order to investigate the effect of stock market shocks during the first wave of the COVID-19 pandemic outbreak.

Having evaluated the characteristic features of precious metals as potential portfolio diversifiers over different investment horizons, we proceed with the second stage of our investigation and consider how these properties changes over time, once again across investment horizons. With this target in mind we follow Fernandez-Macho (2012) and use the decomposed series of precious metals and stock market returns to investigate the dynamic patterns of this relationship by calculating rolling window wavelet correlations between the variables of interest. In this case the co-movement dynamics across different investment horizons (time scales) are analysed over time by using weighted window coefficients. The methodology involves estimating a local movement multiple regression to calculate the correlation maps.

This paper contributes to the literature in several ways. First, it conducts an extensive analysis on the relationship between precious metals and stock markets during the first wave of the COVID-19 pandemic outbreak. Considering precious metals as an asset class, this work complements the literature by offering evidence that the correlation between these commodities and stock markets changes across quantiles and investment horizons. Second, unlike the related literature we extend the analysis to a number of precious metals. Most papers consider only the investment and diversification properties of gold. However, the use and the economic drivers of gold markets are different from those of other markets (see Batten et al., 2010; Beckmann et al. 2015; Baur and McDermott, 2010; Ciner, 2001). Gold is overwhelmingly used for investment, whereas the other precious metals are heavily used in industry. In this respect, the question of the ability of silver, platinum, and palladium to provide portfolio diversification opportunities is still open. In the light of these considerations, this paper adds to the related literature by extending the analysis to a relatively large number of precious metals. The proposed methodological approach is the third contribution of the paper. The main innovation of the suggested procedure is the combination of wavelet analysis with the conditional quantile correlation. Wavelet analysis is a filtering method closely related to Fourier analysis and frequency domain methods that transforms the original data into different frequency components with a resolution matched to its scale. Unlike time series and spectral analysis, which only provides information on time-domain and frequency domain respectively, the wavelet method decomposes the financial time series with respect to both time and frequency domains simultaneously. This allows us to investigate if precious metal returns respond differently over short, medium, and long investment horizons. The analysis of conditional quantile correlation between precious metals and stock markets allows us to investigate the impact of stock market shocks on precious metals returns for all portions of the precious metal probability distribution across a wide number of investment horizons. In this respect, this paper also contributes to an emerging literature that seeks to provide a broad perspective on dependence by modelling the relationship between quantiles (see for example Mensi et al., 2016).

The remainder of this paper is organised as follows: Section 2 presents some background on the role of precious metals in portfolio diversification. Section 3 presents the wavelet quantile correlation procedure and the empirical results. Section 4 presents the dynamic correlation analysis. Section 5 presents the implication for portfolio diversification. Section 6 concludes the remarks.

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1 Note losses due to country-specific risk is also a relevant issue in the portfolio diversification literature. However, the analysis of country-systematic risk is outside the scope of the paper.
2. Literature review

The quest for portfolio diversification benefits has attracted a great number of empirical works on safe-haven and hedging properties of gold. The commodity is often analysed in the literature as a candidate for safe-haven during a period of financial turmoil. For example, McCown and Zimmerman (2006) estimated a CAPM type model and found that gold is a zero-beta asset. Baur and Lucey (2010) were the first to formulate empirically testable definitions for gold as a “hedge” or “safe haven” assets concerning other assets such as bonds or stocks. The authors define a hedge as an asset that is uncorrelated with another asset or portfolio on average, whereas a “safe haven” is defined as an asset that is negatively correlated only in times of market turmoil. (Baur and McDermott, 2010). Capie et al. (2005) investigated the role of gold as a hedge against the dollar and found a negative relationship between gold and other foreign exchange rate haven (see also Upper, 2020; Kaul and Sapp, 2006). Coudert and Raymond (2010) found that gold is negatively correlated with stock markets during bear periods but not in the long run. Similarly, Ciner et al. (2013) reported that gold had a safe haven property against exchange rates in both the United States and the United Kingdom. In addition to gold as a commodity, gold assets such as gold stocks and gold derivatives were also examined as hedge and safe haven by Jaffe (1989) (see also Worthington and Pahlavani, 2007; Pullen et al., 2011).

If the role of gold in portfolio diversification has been widely considered in empirical works, the literature on the properties of the other three precious metals’ potential is still relatively scarce. For example, Hillier et al. (2006a,b) suggested that silver and platinum prices have good portfolio balancing properties since they found negative correlations with stock market indices (see also Aggey-Ampomah et al., 2014; Mackenzie and Lucey, 2013). Further, Daskalaki and Skiadopoulos (2011) found that the returns of silver, platinum and palladium, have low correlations with stock returns. The study by Morales and Andreouso-O’Callaghan (2011) shared that the precious metals markets were less affected by the recent global financial crisis than other major financial markets. Erb and Harvey (2006) and Roache and Rossi (2009) found gold and silver prices as counter-cyclical, implying that other precious metals in addition to gold may also protect investors’ wealth during a period of market turmoil (see also Sensoy, 2013).

From the modelling perspective, most investigators typically adopt the GARCH-type models to analyse the relationship between precious metals and stock markets. For example, Xu and Fung (2005) used a GARCH type model to examine the shock transmission between the US and Japanese markets for precious metals futures contracts. Tully and Lucey (2007) examined the effect of macroeconomic shocks on gold prices with APGARCH models. Using a DCC-GARCH model, Joy (2011) claimed that gold acted as a hedge against the US dollar but provided no evidence of gold being a safe haven for the currency (see also Creti et al., 2013; Salsisu et al., 2020).

The use of wavelet decomposition analysis is relatively recent but is rapidly increasing in the related literature. For example, Bhatia et al. (2020) used a hybrid wavelet-based dynamic conditional correlation approach to study the relationship between precious metals and stock markets of the BRICS and G7 countries in the time-frequency domain. He et al. (2017) suggested to use the multivariate mode decomposition to identify the noise factors in the multiscale domain and forecast the precious metal price movement in order to improve the forecasting accuracy of precious metal prices (see also Das et al., 2018; Yoon, 2017). Oral and Unal (2017) used the wavelet coherence method to analyse the forecast co-movement of precious metals.

On the methodological front, other important studies examined the relationship between precious metals and other assets, like stock and bonds, using quantile-based approaches. These include Baur and Lucey (2010), Mensi et al. (2014), Iqbal (2017) and Adewuyi et al. (2019). An interesting model is suggested in Al-Yanyaee et al. (2020), where a copula quantile-on-quantile regression was used to examine the correlation between precious metals at different quantiles. The procedure allowed the investigators to accommodate all market conditions (i.e. tranquil, normal and turmoil periods) and exploit portfolio diversification opportunities.

3. The wavelet quantile correlation procedure

The proposed WQCOR procedure can easily be carried out in two steps. In the first step the Maximal Overlap Discrete Wavelet Transform (MDWT) is applied to the stock market and precious metal returns in order decompose the series into high-frequency and low-frequency components. In the second step, the obtained filtered series are used as input variables to investigate the portfolio diversification properties of precious metals by calculating the conditional quantile correlation. The two-step procedure to estimate the quantile correlation in the time-frequency domain is described below in more details.

Step 1 The Wavelet Series Expansion

The first step for implementing the WQCOR procedure involves applying the wavelet series expansion to the return series. Wavelet is a technique that decomposes a time series into different short waves that start at a given point in time and end at a given later point in time. In other words, the wavelet approach is a non-parametric method that involves using small wave functions to approximate fluctuations time series to extract information from a sequence of numerical measurements (signals). Broadly speaking, the wavelet decomposition methodology involves applying recursively a succession of low-pass and high-pass filters to the precious metal and stock market series. This process allows separating the high frequency components of the series from the low frequency components (for more details see, for example, Benmound, 2013). Mathematically, the decomposition of the series in different components can be obtained using wavelet transform which is based on two filters. These are respectively called “mother wavelet” and “father wavelet”. The former is useful to capture the detailed (high frequency) parts of the signal whereas the latter gives information on the smooth (low-frequency) part of the signal.

The “father wavelet” (or scaling function) integrates to 1 and is given by

$$\int \varphi(t)dt = 1,$$

whereas the mother wavelet integrates to zero and is given by

$$\int \psi(t)dt = 0.$$  

Since the use of wavelets is a well-established methodology, in this section we only introduce the concepts and definitions useful for our purposes. For an excellent review of the theory and use of wavelets, see Percival and Walden (2000); Gençay et al. (2001).

Let the \( f(t) \in L^2(\mathbb{R}) \) be a function (for \( t = 1, … , T \) the time dimensions can be expressed as a linear combination of a wavelet function

$$f(t) = \sum_k \sum_j \phi_{j,k}(t) + \sum_k \sum_j d_{j,k}\psi_{j,k}(t) + \sum_k \sum_{j,k} d_{j,k}\nu_{j,k}(t) + \cdots$$

(1)

where the orthogonal basis functions \( \phi_{j,k} \) and \( \psi_{j,k} \) are defined as

$$\phi_{j,k} = 2^{j/2} \psi \left( \frac{t - 2^k}{2} \right),$$

$$\psi_{j,k} = 2^{j/2} \psi \left( \frac{t - 2^k}{2} \right).$$

In Eq. (1) the representation \( j \) is the number of multi-resolution components or scales, and \( s_{j,k} \) are the smooth coefficients, and \( d_{j,k} \) are
called the detailed coefficients. They are approximated by the following integrals
\begin{equation}
\begin{split}
s_{jk} &= \int f(t)\varphi_{jk}(t)dt, \\
d_{jk} &= \int f(t)\psi_{jk}(t)dt \text{ for } j = 1, 2, \ldots J.
\end{split}
\end{equation}

The wavelet functions $\varphi_{jk}(t)$ and $\psi_{jk}(t)$ are scaled and translated version of $\varphi$ and $\psi$. The smooth coefficients $2^j$ control the amplitude of the wavelet window so the wavelet function is stretched or compressed to obtain frequency information. Since the scale factor is an exponential function when $j$ gets larger so does $2^j$ and the functions $\varphi_{jk}(t)$ and $\psi_{jk}(t)$ become more spread out and shorter. Therefore, a wider window gives information on the low frequency movements, whereas as narrower windows we get information on the high-frequency movements.

As shown by Bruc and Donoho (1996) if the wavelet coefficients can be approximated by the integral in Eq. (2) and Eq. (3) then a multi-resolution representation in Eq. (1) can be simplified
\begin{equation}
F(t) = S_j + D_j + D_{j-1} + \ldots + D_1 + \ldots + D_j, j = 1, \ldots, J
\end{equation}
where $D_j$ is the $j$-th level wavelet and $S_j$ represents the aggregated sum of variations at each detail of the scale.

In Eq. (1) and Eq. (4) the father wavelet reconstructs the smooth and low-frequency parts of a signal, whereas the mother wavelet function describes the detailed and high-frequency parts of a signal. Therefore, the expression in Eq. (4) provides a complete reconstruction of the time series partitioned into a set of $j$ frequency components so that each component corresponds to a particular range of frequencies.

In the literature several variations of wavelet transform in Eq. (4) have been proposed (see for example Cohen, 1992). In this paper we consider the Maximal Overlap Discrete Wavelet Transform (MODWT). The MODWT has the advantage that the estimated wavelet and scaling coefficients are translation invariant to circularly shifting in the sense that they do not change if the series are shifted in a circular fashion and the smooth coefficients are associated with zero phase filters (for details see Percival and Walden, 2000; Gencay, 2002).

Step 2 The quantile correlation

The second step of the suggested procedure involves using the filtered series obtained from the $j$-level multi-resolution decomposition to estimate the conditional quantile correlation.

Let $X_m$ and $Y_r$ be the set of $m$ precious metals (for $m = 1, \ldots, 4$) and $r$ stock markets (for $r = 1, \ldots, 5$), respectively. We define we define $Q_{1, r}$ be the $r$th unconditional quantile of $Y_r$ and $Q_{1, r} = Q_{0, r}|X_m$ the rth conditional quantile of $Y_r$ on $X_m$. Following Li et al. (2015) we define the quantile covariance as
\begin{equation}
q_{covr}(X_m, Y_r) = \text{cov}\{I(X_m - Q_{1, r} > 0), Y_r\} = E\{o_r(X_m - Q_{1, r})(Y - E(Y))\},
\end{equation}
where the function $o_r(x) = \tau - 1(I(x < 0)$. Accordingly, the quantile correlation between $X_m$ and $Y_r$ is defined as
\begin{equation}
q_{corr}(X_m, Y_r) = \frac{q_{covr}(Y_r, X_m)}{\sqrt{\text{Var}(o_r(Y, X_m))\text{Var}(X_m)}} = \frac{E(o_r(X_m - Q_{1, r})(Y - E(Y)))}{\sqrt{(\tau - \varepsilon)^2\hat{\sigma}^2_r}},
\end{equation}
where $\hat{\sigma}^2_r = \text{Var}(Y_r)$.

Consider the sample $r$th quantile of $Y_{1r}, \ldots, Y_{nr}$ given by
\begin{equation}
\hat{Q}_{r,k} = \inf(k_m : F_r(x_n) \geq \tau),
\end{equation}
where $F_r(x_n) = n^{-1}I(x_n \leq x_m)$ is the empirical distribution function. Given Eq. (5) the sample estimate of the quantile correlation is given by
\begin{equation}
\tilde{Q}_{corr}(X_m, Y_r) = \frac{1}{\sqrt{(\tau - \varepsilon)^2\hat{\sigma}^2_r}} \frac{1}{\sum_{i=1}^{r} a_i (X_m - \hat{Q}_{r,k})(Y_r - \hat{\mu}_r)}. \end{equation}

where $\hat{\mu}_r = n^{-1}\sum_{i=1}^{r} Y_r$ and $\hat{\sigma}^2_r = n^{-1}X_r - \hat{\mu}_r)^2$. Li et al. (2015) show that, under regular conditions, the expression in Eq. (6) is a consistent estimator of the quantile correlation and that $\sqrt{n}(\tilde{Q}_{corr}(X_m, Y_r) - q_{corr}(X_m, Y_r))$ converges in distribution to a $N(0, \Omega)$.

We can use the information provided by the WQCOR procedure to investigate if gold and the other precious metals can be used as asset classes by investors to balance investment portfolios during times of high market volatility. With this target in mind, we follow the definition adopted in Baur and McDermott (2010) (see also Baur and McDermott, 2010) and define an asset as a safe-haven if it is uncorrelated or negatively correlated with another asset or portfolio in times of extreme market movements. Alternatively, an asset is defined as a hedge if it is uncorrelated or negatively correlated with another asset or portfolio on average. The crucial distinction between these two features is that dependence is required to hold under extreme market movements for a safe haven, whereas, for a hedge, it must do so on average. Baur and McDermott (2010) draw a distinction between strong and weak hedges and safe havens on the basis of the negative or null value of the correlation, respectively.

Accordingly, for the $m$ precious metal to be classified as safe haven the estimated correlation coefficient between the lower quantiles of the $m$ precious metal and the $r$ stock market has to be negative. Similarly, to be classified as hedge, the estimated correlation coefficients among the median quantiles of the $m$ precious metal and the $r$ stock market have to be negative or zero. We can thus formulate two hypotheses to determine whether the $m$ metal can serve as a hedge or as safe haven against stock prices:

Hypothesis 1 $X_m$ is a safe-haven if $q_{corr}(X_m, Y_r) < 0$ for $\tau < t_0$

Hypothesis 2 $X_m$ is a hedge if $q_{corr}(X_m, Y_r) \leq 0$ for $t_0 < \tau \leq t_1$

Where $t_0 = 0.3$ and $t_1 = 0.6$. Note in Hypothesis 2 unlike the definition in Baur and McDermott (2010) where the media is considered, we slightly modify the criterion by taking into consideration the quintiles around the median. The rational of doing so is that the second moment of a distribution is notoriously affected by outliers, whereas the median does not have this drawback and seems to be more appropriate for our application.

Note also that Hypothesis 1 is in practice a test on the measure of dependence between the upper tail of $X_m$ and the lower tail of $Y_r$ which is the probability.

Hypothesis 1 $X_m$ is a safe haven if $\lim_{t_0 \rightarrow 1} P(Y_r > Q_{0, r}|X_m) < Q_{y_r(1-\tau)}$.

Likewise.

Hypothesis 2 $X_m$ is a hedge if $\lim_{t_1 \rightarrow 1} P(Y_r > Q_{0, r}|X_m) < Q_{y_r(1-\tau)}$.

In addition to Hypotheses 1 and 2, extreme upward market movements of $X_m$ and $Y_r$ may also be of interest, since in this case in this case investors may want extreme upward movements to be positively correlated. Therefore, the following hypothesis may be tested.

Hypothesis 3 $X_m$ is a strong hedge if $q_{corr}(X_m, Y_r) > 0$, for $\tau > t_1$.

Or alternatively.

Hypothesis 3 $X_m$ is a strong hedge if $\lim_{t_1 \rightarrow 1} P(Y_r < Q_{0, r}|X_m) < Q_{y_r(1-t_1)}$.

Under Hypothesis 3, $X_m$ is a strong-hedge if high positive returns are correlated to high positive returns of $Y_r$. Note that according to the efficient market hypothesis positive correlation of extreme upward movement between two assets should not last long since it should be impossible to beat the market consistently on a risk-adjusted basis and market should react to new information (see for example Fama, 1970).
### 3.1. Data and empirical results

The data used in this study are daily stock market returns from 20th December 2019 to 15th July 2020 of four major emerging economies, namely Brazil, Russia, India, China. The stock market indexes we consider are the IBV index for Brazil, IMOEX for Russia, NIFTY50 for India, SSE for China and the S&P index for the U.S. as a proxy of the largest economy in the world. In addition, the dataset includes prices for four precious metal indexes namely gold, silver, platinum, and palladium. Note that the data under consideration are denominated in US dollars to facilitate the comparison between stock market indices and safe-haven assets. Following the literature, stock returns are calculated as the difference of the logarithm of the price index.

Table 1 A-B reports the estimated conditional quantile correlations

<table>
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<th>SSE</th>
<th>IMOEX</th>
<th>IBV</th>
<th>NIFTY</th>
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</tr>
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<td>0.025</td>
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</tr>
<tr>
<td><strong>D5 (32-64 days)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>τ</td>
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<td>0.071</td>
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</tr>
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<tr>
<td></td>
<td>0.4</td>
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<tr>
<td><strong>Silver</strong></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td><strong>D1 (2-4 days)</strong></td>
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<td></td>
</tr>
<tr>
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<td>0.081</td>
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</tr>
<tr>
<td><strong>D3 (8-16 days)</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>τ</td>
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<td>-0.018</td>
<td>-0.034</td>
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<tr>
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<td>-0.013</td>
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<tr>
<td></td>
<td>0.4</td>
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<td>-0.123</td>
<td>-0.123</td>
<td>-0.082</td>
</tr>
<tr>
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<td>0.046</td>
<td>0.100</td>
<td>-0.100</td>
<td>-0.100</td>
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<td>0.082</td>
<td>0.082</td>
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</tr>
<tr>
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<td>0.066</td>
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<tr>
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<td>0.051</td>
<td>-0.139</td>
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<tr>
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<td>0.034</td>
<td>-0.219</td>
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<tr>
<td><strong>D5 (32-64 days)</strong></td>
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</tr>
<tr>
<td>τ</td>
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<td>0.050</td>
<td>0.135</td>
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</tr>
<tr>
<td></td>
<td>0.4</td>
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<td>0.096</td>
<td>0.110</td>
<td>0.138</td>
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<tr>
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<td>0.083</td>
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</table>

Note: The table reports the estimated quantile correlation coefficients between precious metals and stock market indexes by scale. To obtain the wavelet coefficients at each scale, the Daubechies extremal phase wavelet filter of length 8 is applied. The method of estimating the quantile correlation is defined in Eq. (6).
between the $X_m$ precious metal and the $Y_t$ stock market for each time scale. The decomposition of the $r$-stock and $m$-precious metal returns were obtained by applying the MODWT with the Daubechies compactly supported least asymmetric (LA) wavelet filter of length $L = 6$ (Daubechies, 1992). This filter has been successfully used in several empirical studies to capture the characteristic features of the financial data (see Gallegati, 2012 and the references therein).

Note that for all families of Daubechies compactly supported wavelets the level $j$ wavelet coefficients are associated with changes at the $\lambda_j = 2^{-j}$. However, since the MODWT utilizes approximate ideal bandpass filters with bandpass given by the frequency interval $[2^{-(j+1)}, 2^{-j})$ for $j = 1, \ldots, J$, inverting the frequency range we have that the corresponding time periods are $[2^j, 2^{j-1})$ time units (Fernandez-Macho, 2012).

### Table 1B

#### Quantile correlation.

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\Delta$ (2-4 days)</th>
<th>$\Delta$ (4-8 days)</th>
<th>$\Delta$ (8-16 days)</th>
<th>$\Delta$ (32-64 days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$ (8-16 days)</td>
<td>SSE</td>
<td>IMOEX</td>
<td>IBV</td>
<td>NIFTY</td>
</tr>
<tr>
<td>$\Delta$ (32-64 days)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta$ (64-128 days)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The table reports the estimated quantile correlation coefficients between precious metals and stock markets indexes by scale. To obtain the wavelet coefficients at each scale, the Daubechies extremal phase wavelet filter of length 8 is applied. The method of estimating the quantile correlation is defined in Eq. (6).
2012). In our case since we have 20 daily data for each series per month, we set the scale \( j = 6 \), the highest frequency component \( D_1 \) represents short-term variations corresponding wavelet coefficient for intraweek scales 2–4 days, \( D_2 \) accounts for variations at a time scale of 4–8 days, near the working days of a week. Similarly, \( D_3 \) and \( D_4 \) components represent the variations at time scale of 8–16 (approximately fortnightly) and 16–32 days (approximately monthly scale), respectively. Finally, \( D_5 \) and \( D_6 \) components represent the long-term variations at time scale of 32–64 (approximately monthly to quarterly scales) and 64–128 days (quarterly to biannual scales). \( S_0 \) is the residual of the original signal after subtracting \( D_1, D_2, D_3, D_4, D_5 \) and \( D_6 \).

In Table 1 A-B, the correlations between precious metals and stock returns are reported. The second column reports the \( r \)-quantile, whereas the metal and the stock market under consideration are reported in the first column and the first row, respectively. Note that in Table 1 A-B, the quantile correlation does not enjoy the symmetric property, this is because the measure of tail dependence through conditional quantiles is dependent on the order of the variables and thus on the non-exchangeability between the variables. This implies that it may be possible that the conditional distribution of the \( Y_i \), stock market given \( X_0 \), precious metal displays tail dependence, whereas \( X_0 \) does not display tail dependence.

Considering now Hypothesis 1, looking at the top part of Table 1A it appears that for most stock markets gold is better able to serve as safe-haven than for hedging since most of the estimated correlations in the lower quantiles are negative, especially in the medium long scales (investment horizons). It is interesting to note that the estimated correlation coefficients are rather stable across different investment horizons since the negative sign persists up to time scale D6. Coming to silver, the picture changes since it appears that the ability of silver to act as safe-haven is more scale dependent with medium, low-frequency bands (long horizons) performing better than high-frequency ones (short horizons).

With regard to platinum and palladium in Table 1B there is evidence that they both enjoy the safe-haven property since correlations in the lower quantile are mostly negative. It also appears that the relationship between returns is not scale dependent since the correlations are negative no matter the frequency scale under consideration.

Coming to Hypothesis 2, in Table 1A it is clear that gold may offer diversification opportunities to investors mainly in the medium or long run horizons, since the estimated correlation coefficients for D4-D6 are negative or close to zero. Looking at silver, we observe that correlations are more country and scale-dependent since the estimated correlation coefficients are negative for IMOX in scales D2-D5, however IBV enjoys negative coefficients for scales D3 and D4. By contrast, NIFTY has a negative correlation throughout all the time scales, but D1 and D5. The estimation results for the other precious metals indicate that platinum offers a good hedge opportunity for some stock markets such as IBV and SEE where correlations are negative for all time scales (IBV) or all but D1 and D4 (SSE). For other stock markets the situation is more mixed with platinum acting as a hedge for the IMOX stock index for scales D3-D5 and the S&P and NIFTY indexes with only two-time horizon opportunity.

Finally, the performance of palladium is, once again market-dependent, with the precious metal performing very well for some markets such as SSE and IMOX, and poorly for others such as NIFTY where correlations are negative for the medium time horizons only.

As far a Hypothesis 3 is concerned, this hypothesis corresponds to days of market exuberance where both metals and stock markets enjoy strong positive returns. Therefore, in this case investors are interested in the positive correlation since negative correlations would induce a loss for portfolio returns. Looking at the results in Table 1A, it appears that gold offers good prospects since correlations are mostly positive, no matter the time scale under consideration. The situation for the other metals is more mixed with metals offering good opportunities for some markets, but not for others.

4. Dynamic wavelet correlation

The results in Table 1 A-B are informative about the joint behaviour of precious metals and stock markets in different time scales. However, it is also of interest to look at the dynamics of the correlation since it is well known that correlations change over time. To investigate this issue we follow Fernandez-Macho (2018) and use the time-localized multiple regression model to estimate the time varying correlation between precious metals and stock markets. The method allows to calculate the set of multiscale correlations over time and across different scales by estimating a series of windowed wavelet coefficients. In particular, let \( Z = (X_{it}, Y_t) \) be a realization of a multivariate stochastic process, and \( Z_{-t} = Z_{\{z_i\}} \) for some \( z_i \in Z \). For a fixed \( s \in \{1, \ldots, T\} \), Fernandez-Macho (2018) suggest to minimize a weighted sum of the squared errors

\[
S_s = \min \sum \theta(t-s)[f(Z_{-t}) - \zeta(t,s)]^2,
\]

where \( f(Z_{-t}) \) is a linear function and \( \theta \) is a given moving average weighted function. The local weighted least squared approximation around \( s \) can be written as

\[
f(Z_{-t}) = M \beta_s,
\]

where \( M_t = Z_{-t} - Z_{-\tau} \). The \( \beta_s \) can therefore be estimated by calculating

\[
\hat{\beta}_s = \left[ \sum_{t} \theta(t-s)M_t, M_{t-1}\right]^{-1} \sum_{t} \theta(t-s)M_t, \zeta(t,s),
\]

Letting \( s \) moving overtime, for the local regression in Eq. (7) local coefficient of determination can be calculated from the weighted sum of the local squared residuals.

In analogy with the classical regression model, the wavelet local multiple correlation coefficient at each wavelet scale can be estimated as follows:

\[
\hat{r}_{M,s}(\lambda) = \text{Corr}\left(\theta(t-s)^{1/2} \omega_{l_1}, \theta(t-s)^{1/2} \omega_{l_2}\right)
\]

where \( \omega_{l_1} \) are the wavelet coefficients chosen to maximise \( \hat{r}_{M,s}(\lambda) \) and \( \omega_{l_2} \) are the fitted values in Eq. (8) (for more details, see Fernandez-Macho, 2018) the joint behaviour of precious metals and stock markets in times of extreme market movements.

In Fig. 1, the correlation patterns between the returns of the precious metals under consideration and the stock market indexes are presented in a time-frequency domain on a scale by scale basis. Therefore, in Fig. 1 the correlation coefficients are calculated daily for each pair of stock markets returns and precious metals. For ease of interpretation, the left-hand horizontal axis is transformed to show the number of days in which the scale moves from low to high wavelengths. The heat maps indicate the increasing strength of the correlation among the stock market indexes as they move from blue (lowest correlation) to red (highest correlation).

Looking at the results in Fig. 1, it appears that the time horizon under consideration is quite an important feature when it comes to evaluating the performance of precious metals as portfolio stabilizers in times of market distress. The correlation patterns are also different across stock markets with some markets performing better than others.

For example, for the Chinese stock market, we can infer from the light/dark blue colour in Fig. 1 that gold and palladium have good diversification opportunities properties for long portfolio horizons (i.e. above 64 days), D6 scale (note that a contagion effect emerges for the scale D6 during the peak of the COVID-19 outbreak between January 2020 to March 2020 as highlighted by the red colour in Fig. 1). However, the estimated negative correlations suggest that silver and platinum are better choices for short-medium/medium portfolio horizons. Looking at the Russian stock market it appears that precious white metals offer good portfolio stabilizer opportunities, especially in short-
medium scales since the light/dark blue colour indicates low or negative estimated correlations for these metals. On the contrary, gold returns show signs of contagion with the IMOEX stock market returns as the estimated correlations appear to be positive for most investment horizons. Coming now to the Brazilian stock market, the picture changes since all the precious metals under consideration perform badly for long-
Fig. 1. (continued).
horizon investments, but offer good portfolio stabilizer opportunities in the short-medium time scales. Interestingly, contrary to the other stock markets, gold, silver and to some extent platinum, offer good portfolio opportunities in the long-horizon time scales for the Indian stock market, but the same is not true for palladium where short-medium time scales have negative estimated correlation coefficients. Finally, in the US the S&P stock market shows some similarities with the IBV stock market, with platinum and palladium performing well for the short-medium investment horizons (investment horizon up to 64 days).

Overall, the findings in Fig. 1 suggest two main results: i) There is evidence that precious metals offered portfolio diversification opportunities during the COVID-19 health crisis. However, these opportunities are market-dependent with some stock markets performing better than others; ii) The diversification property is scale-dependent since negative correlations have been observed in the short-medium frequency for platinum and palladium for most stock markets, whereas the opposite is true for gold and silver where spells of negative correlation are observed in the long-run only. This result may be explained by the fact that economic drivers for gold and silver are different from the other metals (see for example Batten et al., 2010). The demand for the former metals is dominated by their role as monetary assets as well as commodity, whereas the demand of the latter is mainly driven by their industrial use. The uncertainty triggered by the COVID-19 pandemic may have played a role in shaping the correlation patterns observed in this study. In this respect, Baur and Glover (2012) argue that the demand for gold is heavily affected by market sentiment. For this reason, investors’ behaviour has the potential to undermine and possibly destroy the safe haven property of gold when risk aversion suddenly and sharply increases since this behaviour fundamentally change the transmission channel of risk between assets.

5. Implications for portfolio construction

The analysis in the previous sections provided several insights on the dependence structure between stock markets and precious metals. In particular, the analysis in Section 3 allowed the investigators to answer the following question: How does the correlation of the r-quantile of the m-metal (conditional on the r-stock return) change when the returns of r-stock market change? In this section, we are interested in answering the following question: What are the implications for risk management and portfolio construction strategies?

To examine this issue we compute optimal portfolio weights and hedge effectiveness of m precious metals for the r stock index under consideration using the conditional variance and covariance estimates obtained using the WQCOR procedure. With this target in mind we estimate the optimal portfolio weights to evaluate the optimal proportion of the precious metals and the equities that should form a rational investor’s portfolio. To evaluate the optimal portfolio weight, we propose a quantile-based and horizon dependent approach which is a variation of the method suggested in Kroner and Ng (1998). Namely, the investor in the m-metal, willing to hedge against adverse price movements in the stock r-portfolio without short selling, can decide their portfolio according to the following formula:

\[
\sigma_{mr} = \frac{\text{Var}(X_m) - \text{Cov}(X_m, Y_r)}{\text{Var}(X_m) - 2\text{Cov}(X_m, Y_r) - \text{Var}(Y_r)},
\]

where \(\sigma_{mr}\) is the weight of the m-metal in a one-dollar portfolio of \(X_m\) and \(Y_r\) at time \(t\), index \(\text{Var}(X_m)\) and \(\text{Var}(Y_r)\) are the conditional variance of \(X_m\) and of the variance of \(Y_r\), respectively, obtained the WQCOR procedure and \(\text{Cov}(X_m, Y_r)\) is the conditional covariance between metal \(m\) and equity index \(r\). Note that in Eq. (9) the subscripts for the r-quantile and the j time scale (investment horizon) have been omitted to improve clarity. The weight of \(X_m\) is calculated as \((1 - \sigma_{mr})\).

Table 2 reports the summary statistics of the portfolio weights that were computed by applying the Kroner and Ng (1998) approach for the optimal weights in portfolio selection. To save space, only the results for gold, silver, SP, and IBV are presented. Namely, silver is considered as representative of the white metals and the IBV index as a benchmark for the BRIC stock markets. Also, the results relate to the optimal portfolio weights for time scales D2 and D5 only. These scales are chosen as representative for short investment and long investment horizons, respectively. Note that the optimal weights for time scales in similar classes, (i.e. D1 for short and D4 for long horizon) were similar to those reported in Table 2 and the results have been omitted, but available on request. In the second column, the relevant quantiles are described, so that r ≤ 0.3 corresponds to the optimal portfolio weights in case of extreme negative returns in stock markets under consideration. As before, the middle quantiles correspond to hedging during “normal” periods and the top quantiles are related to the optimal portfolio weights in case of extreme positive returns in stock markets.

Looking at the results from Table 2 it appears that the optimal choice

<table>
<thead>
<tr>
<th>Quantile</th>
<th>Gold</th>
<th>Silver</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>St. Dev</td>
</tr>
<tr>
<td>5&amp;P</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r ≤ 0.3</td>
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<td>0.114</td>
</tr>
<tr>
<td>0.3 &lt; r ≤ 0.6</td>
<td>0.507</td>
<td>0.363</td>
</tr>
<tr>
<td>0.6 &lt; r ≤ 0.9</td>
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<td>0.214</td>
</tr>
<tr>
<td>IBV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r ≤ 0.3</td>
<td>0.621</td>
<td>0.157</td>
</tr>
<tr>
<td>0.3 &lt; r ≤ 0.6</td>
<td>0.437</td>
<td>0.189</td>
</tr>
<tr>
<td>0.6 &lt; r ≤ 0.9</td>
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<td>0.134</td>
</tr>
<tr>
<td>D5</td>
<td></td>
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</tr>
<tr>
<td>r ≤ 0.3</td>
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</tr>
<tr>
<td>0.3 &lt; r ≤ 0.6</td>
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<td>0.189</td>
</tr>
<tr>
<td>0.6 &lt; r ≤ 0.9</td>
<td>0.291</td>
<td>0.111</td>
</tr>
<tr>
<td>IBV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r ≤ 0.3</td>
<td>0.751</td>
<td>0.341</td>
</tr>
<tr>
<td>0.3 &lt; r ≤ 0.6</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>0.6 &lt; r ≤ 0.9</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
for the gold-S&P portfolio for the investment horizon 4-8 days for the \( r \leq 0.3 \) quantile, is 0.577. This indicates that for a $1 portfolio, nearly 60 cents should be invested in gold and 40 cents in S&P stock market index. In more normal periods this proportion should be reduced roughly to 50 cents in each asset for the same investment horizon. For longer investment horizons, this percentage reduces considerably, no matter the quantile under consideration. Considering now the optimal weights for the silver-S&P portfolio the average weights are lower in general for the D2 time scale. This result is in line with the conditional correlation coefficients reported in Table 1 A-B that are in average higher for gold-S&P than silver-S&P. Coming now to the D5 scale only the optimal weights for the lower quantiles are reported, since according to the results in Table 1 A-B the correlations for this time scale are positive, therefore silver does not constitute a diversifier for the Silver-S&P stock portfolio. Looking now at the optimal weights for the Gold-IBV portfolio the weight is higher for the lower quantiles but lower for the other quantile. Silver does not look a good diversifier in the long horizon in Table 1 A-B, therefore the optimal portfolio weights have been omitted.

6. Discussion

Before concluding this section a question is in order: What do we learn about the relationship between stock market and precious metals? The results in Table 1 A-B illustrates that the multiscale relationship can be a useful tool for portfolio diversification. Market participants constitute a very heterogenous group and make investment decisions over different horizons. For example, intraday traders have diverse objective from hedging strategists, international portfolio managers, or large multinational corporations. Since investors have different time horizons, it is natural for them to seek to minimise portfolio idiosyncratic risk at a given time scale. For example, even in condition of extreme market distress such as COVID-19 outbreak, positive correlations in the short-run between precious metals and stock markets may not be important to their investment goals to investors with long-run time horizons, such as pension funds for example. However, most previous empirical studies focus on two-scale analysis, namely the short-run and long-run periods. The proposed WCOR procedure allows the investigator to pre-process financial time series of interest using the multiscale decomposition to analyse the conditional quantile correlations at different investment horizons. In this respect, the wavelet may act as a “lens” enabling the investigator to unveil characteristic features of time series that would not be observable using traditional econometric methods.

We are not the first authors to use wavelets to analyse the relation between financial time series variables (see for example Al-Yahyae, 2019). However, most of the related literature makes use of discrete and continuous wavelet transforms to calculate the tail dependence or the wavelet correlation coefficient between financial series. The WQCOR procedure allows to measure the quantile dependence between time series for each quantile. Therefore, it allows to capture how the relationship changes in each state of the market (e.g. bearish, bullish, or normal). The suggested procedure is in the spirit of Meni et al. (2016) (see also Xu et al. 2020) where quantile regression models are estimated from the decomposed series obtained using a wavelet analysis. It can be shown that

\[
q_{\text{COR}}(X_\alpha, Y_r) = \text{sign}(\beta_{r \times r}(\tau)) \sqrt{\beta_{r \times r}(\tau)\beta_{r \times r}(\tau)},
\]

where \( \beta_{r \times r}(\tau) \) are the regression coefficients of the quantile regression of \( X_\alpha \) on \( Y_r \), and \( \beta_{r \times r} \) are the estimated coefficient of the reciprocal quantile regression (see Choi and Shin, 2018). Therefore, taking the geometric mean of the two \( q \)-quantile regressions delivers similar results to those obtained from the WCOR procedure. However, the proposed procedure is computationally less cumbersome to estimate. Moreover, the upper and the lower tail parameters are notoriously difficult to estimate by maximum likelihood. This is due to the fact that there are relatively fewer observations in the tails of the conditional distributions of returns. The problem can be easily avoided by directly estimating the correlation coefficient between quantiles.

From the results in Fig. 1 and Table 1 A-B it is clear that precious metals may play an important role in balancing portfolios. However, correlations not only change over quantiles and investment horizons, but also over time. Therefore, it is crucial for investors to consider the anticipated holding period of precious metals assets in their portfolios. From the econometric point of view, a growing number of empirical applications in the field have found evidence of consistent nonlinear dependencies. For example, Choudhry et al. (2015) investigate the nonlinear dynamic co-movements between gold returns and stock market returns and found evidence of nonlinear feedback effect among the variables during the during the global financial crisis period (Kyrtsou et al., 2006). The evidence of nonlinearity corroborates the use of the MODWT since wavelet transforms are robust to regime shifts. One may argue that the time-localized wavelet multiple regression approach is particularly fit for purpose, since the methodology is robust to changes in the dependence structure of the stochastic processes found under investigation, such as non-stationarity spells of processes for example or any type of changes in the dependence structure of the stock market series that we may observe during periods of extreme market distress such as the COVID-19 outbreak. Failing to account for the characteristic feature of the series would result in a less than optimal investment strategy for investors. Looking forward, it would be interesting to use factor models on the decomposed series of precious metals returns and apply exogenous variables that might explain the systematic risk of a portfolio on a scale-by-scale basis. This would allow us to investigate the effect of systematic risk factors on portfolio diversification on different investment horizons.

7. Conclusion

In this study we investigate the role of precious metals such as gold and silver in portfolio diversification assets during the early period of COVID-19 pandemic. With this target in mind, we propose a novel approach to investigate the dependence structure between precious metals and the stock market for different investment horizons by using wavelet decomposition prior calculating the quantile correlation coefficient. The suggested WCOR procedure identifies the quantiles of metal and stock returns where the negative correlation allows to achieve maximum benefit from portfolio diversification for each given investment horizon. At the same time, for each investment horizon, the procedure also allows to spot the quantiles of stock and metal returns at which the correlations are positive, thus providing an opportunity for portfolio managers to make informed decisions about when they should avoid going long or short on both the assets under consideration. This gives investors flexibility about the choice of the time and at what investment horizon they should enter the market.

The estimated results reveal that precious metals can be successfully used to balance portfolios even in periods of extreme market distress. In particular, the WCOR procedure suggests that gold can act as a safe-haven especially for medium-long run investment horizons (investment strategies 8–16 days and longer), although some evidence of positive quantile correlation between the lower quantiles of the yellow metal and the quantile of IMOEX, NIFTY, and IBV market returns is found for horizons between 4 and 8 days. Silver is also found to performs better as a medium-long run safe haven since pockets of short-run positive correlation between the lower quantile are found for horizons shorter than 8 days. Looking at the median correlations it appears that gold is better able to act as a hedge for most stock markets for the short-medium investment horizons, whereas silver better serves as a hedge in medium run horizons. Perhaps the most important finding of the paper is that gold and silver, which are the most well-known precious metals, are not the only available commodities to investors. The quantile correlation...
analysis has revealed that platinum and palladium in several instances overtake the former metals in terms of safe-haven and hedging properties. In this respect, our results are in line with Bredin et al. (2015) where it was found that gold and gold stocks have been shown to have low levels of correlation with equity indices, highlighting their role as a diversifier (see, for example, Chua et al., 1990; Hillier et al., 2006a,b).

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