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Cristian Bartolucci
Ignacio Monzon

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Frictions Lead to Sorting: a Partnership Model with On-the-Match Search*

Cristian Bartolucci and Ignacio Monzón†

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Abstract

We present a partnership model where heterogeneous agents bargain over the gains from trade and search on the match. Frictions allow agents to extract higher rents from more productive partners, generating an endogenous preference for high types. More productive agents upgrade their partners faster, therefore the equilibrium match distribution features positive assortative matching. Frictions are commonly understood to hamper sorting. Instead, we show how frictions generate positive sorting even with a submodular production function. Our results challenge the interpretation of positive assortative matching as evidence of complementarity.

JEL Classification: C78; D83; J63; J64

Keywords: Assortative matching; Search frictions; On-the-match search; Bargaining

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†Collegio Carlo Alberto, Via Real Collegio 30, 10024 Moncalieri (TO), Italy. Emails: cristian.bartolucci@carloalberto.org and ignacio@carloalberto.org.

1. Introduction

Markets with two-sided heterogeneity are prevalent. In labor markets, firms and workers typically differ in their characteristics, quality and ability. The same is true in other markets, such as the marriage market and the market for CEOs. The evidence suggests that better CEOs sort into better corporations (Parrino [1997]), that there is positive assortative mating in the marriage market (Mare [1991]), and that more productive employees work for better firms (Bartolucci and Devicienti [2013]). Traditionally, positive assortative matching has been interpreted as evidence of complementarity in the production function. In this paper we argue that frictions are a natural reason for positive assortative matching to arise, even in the absence of complementarity in production.

In the presence of frictions higher types become more appealing. While in frictionless markets payoffs reflect individual contributions, the division of output becomes more even when it takes time to find a partner. To see this, assume that agents are infinitely impatient (or frictions infinitely strong), and therefore outside options are zero. In this simple case, the gains from trade are equal to the production of the match. Under standard bargaining, both agents receive an equal share of the gains from trade, so agents receive a constant fraction of the match's output. When frictions are strong enough, it is the total production of the match, rather than individual contributions, that shapes payoffs and preferences over partners. Production is increasing in the partner's type, so an endogenous preference for better types arises. Complementarity in production only plays a secondary role.

A preference for high type partners leads to positive sorting when agents are allowed to search while matched. When agents on both sides of the market can replace their partners, more desirable agents upgrade partners faster. In this way, a preference for high type partners makes high type agents climb the ladder of partners faster. Therefore, as in Lentz [2010], the distribution of matches features positive sorting. If instead agents are not allowed to replace their partners (like in Shimer and Smith [2000] and Atakan [2006]), a preference for high types does not translate into positive sorting.[1]

Match-to-match transitions are pervasive in most developed economies. Fallick and Fleischman [2004] estimate that at least half of all new employment relationships result from job-to-job transitions. On the firm side, Albak and Sørensen [1998] and Burgess, Lane, and Stevens [2000] present empirical evidence of replacement hiring (see Kiyotaki and Lagos [2007] for a discussion). In the market for CEOs, Parrino [1997] finds that the availability of a strong outside candidate is an important consideration in the decision to replace a poor CEO. Murphy and Zabojnik [2006] report that a large proportion of managers were hired from another firm. Stevenson and Wolfers [2007] find that remarriage is one of the main determinants of divorce.

We present a partnership model with transferable utility where agents search on the match.[2] A matched agent who finds a new partner can dissolve the current match and form a new one. After dissolving a match, the agent bargains with the new partner without the possibility of returning to the previous one. Our bargaining protocol prevents agents from exploiting the presence of multiple suitors to raise their payoffs. This timing makes preferences over partners simple: the value of the match to an agent depends only on her *current* partner's type. In some markets (like the one for academic economists) counteroffers are common practice. However, this is not the norm in most markets (see Mortensen [2005]). In Section 5.2 we modify the bargaining protocol to allow for renegotiation and show how frictions can lead to positive sorting in this case, also without productive complementarity.

Allowing agents to search on the match adds an extra layer of difficulty to the bargaining problem: the surplus from the match depends on the bargaining outcome. Patient agents face a trade-off between per-period payoff and expected du-

[1]In Shimer and Smith [2000], a preference for high type partners implies acceptance sets with non-increasing lower bounds. Any partner accepted by an agent is also accepted by better agents. See Section 5.1 for an in-depth discussion.

[2]In our framework match payoffs are not exogenously given, but rather endogenously determined through bargaining. Hence, in our environment utility is transferable (Smith [2006]).

ration of the match: higher wages paid to a worker reduce the firm's per-period profits, but they decrease the likelihood that the worker quits. In fact, a higher wage may increase the value of the match both to the worker and the firm. As highlighted by Shimer [2006], bargaining sets are not necessarily convex and therefore the standard axiomatic Nash Bargaining is not applicable to this setup.

We present a solution for axiomatic bargaining when both sides can leave the match if they find a preferred option. Bargaining sets do not satisfy Nash's axioms [1950]. However, we show that bargaining sets are compact. We follow a modified version of Nash's axioms proposed by Kaneko [1980]. Kaneko shows that for compact bargaining sets the solution is exactly as in Nash [1950]: it selects the outcome which maximizes the product of agents' individual surpluses.

In our benchmark model there are two agent types (low and high). This simple model is rich enough to illustrate the trade-offs that agents face with on-the-match search. We show that several different equilibria can arise, depending on the degree of complementarity in production, agents' patience and the degree of frictions in the market. Each possible equilibrium induces a pattern of sorting. We show that a preference for the high type is sufficient for positive assortative matching to arise in our model. We fully characterize this two type model. We provide necessary and sufficient conditions for existence and uniqueness of an equilibrium featuring an endogenous preference for the high type. We finally present necessary and sufficient conditions for positive assortative matching.

Our intuition extends to the case with any finite number of types. We show that an equilibrium where agents endogenously prefer higher types arises as agents become impatient or frictions large. Moreover, this is the unique equilibrium. Next, we provide numerical examples with parameter values in line with the literature where agents endogenously prefer higher types. Both with modular and submodular production functions there are equilibria where matching is positively assortative.

The literature on assortative matching mostly focuses on how complementarity in production affects the allocation of workers to firms. In Becker's seminal partnership model [1973], a supermodular production function is necessary and sufficient for positive assortative matching. This is not true in markets with frictions.[3] When it takes time to find a partner, agents are selective only if complementarity in production compensates the cost of waiting. Therefore, the conventional wisdom is that stronger frictions require stronger complementarity in production for positive assortative matching to arise. Our paper highlights a different role for frictions: they modify preferences over partners. Thus, frictions can generate positive assortative matching even with a submodular production function. Our results challenge the interpretation of sorting as evidence of complementary in production.

Policy recommendations differ when sorting results from frictions, rather than from complementarity. Consider the linear production function case. There, positive sorting can only arise because of frictions. However, if sorting is interpreted as evidence of complementarity in production, the standard policy recommendation is to subsidize agents to wait until they find their preferred partner (see Acemoglu and Shimer [1999a]). Now, since production is linear, a different distribution of matches does not change the aggregate production of the economy and moreover search is costly in terms of forgone output. Then, such a program would be welfare detrimental.

The rest of the paper is organized as follows. In the next section we present the model, describe bargaining sets with on-the-match search and present our notion of equilibrium. In Section 3 we solve the two type case. We provide a full characterization of all equilibria in this simplified setting. Section 4 shows how our results extend to the case with any number of types. Section 5 relaxes some of the main assumptions of our baseline model. We show that frictions lead to sorting

[3] There is no positive assortative matching in Shimer and Smith [2000] for modular and slightly supermodular production functions (see Section 5.1 for an in-depth discussion). In Atakan [2006], whenever the explicit cost of search is high and complementarity weak, random sorting arises in equilibrium. In Eeckhout and Kircher [2010] root-supermodularity is necessary and sufficient for positive assortative matching.

in cases where search on the match is less efficient than out of the match; in cases with renegotiation; and in the constrained efficient allocation chosen by a central planner. Section 6 concludes.

2. The Model

Consider a continuous time, infinite horizon stationary economy, populated by infinitely lived, risk neutral agents. There is a unit mass population of heterogeneous agents denoted by their fixed type $x \in X$, where X is a finite ordered set of possible types. All types are present in equal proportion in the population.

Agents can be either matched or unmatched. Transitions between states occur due to exogenous destruction and match-to-match transitions. Matches are exogenously destroyed at rate δ and meetings occur at rate ρ. Agents discount the future at rate $r > 0$.

A match produces a flow of output $f(x, y) : X^2 \to \mathbb{R}_+$. The production function is strictly increasing in both arguments and symmetric: $f(x, y) = f(y, x)$. Unmatched agents produce zero. Until Section 4 we assume that there are two types: $X = \{\ell, h\}$. In this case $f(\ell, \ell) = 2\ell$, $f(h, h) = 2h$, and $f(h, \ell) = f(\ell, h) = F$, with $0 < 2\ell < F < 2h$. Parameter F captures the degree of complementarity in production. A modular production function has $F = \ell + h$, a supermodular one has $F < \ell + h$, and $F > \ell + h$ corresponds to the submodular case.

The steady state distribution $e(x, y) : \{\ell, h\} \times \{\varnothing, \ell, h\} \to \left[0, \frac{1}{2}\right]$ specifies the number $e(x, \varnothing)$ of unmatched x-type agents and the number $e(x, y)$ of x-type agents matched to agents of type $y \in \{\ell, h\}$. Since in the population there are as many low as high productivity agents, $\sum_{y \in \{\varnothing, \ell, h\}} e(x, y) = \frac{1}{2}$ for $x \in \{\ell, h\}$. We allow *both* matched and unmatched agents to meet potential partners (who *also* themselves may be matched or unmatched). Any agent, regardless of type and match status, meets an agent $x \in \{\ell, h\}$ currently matched to $y \in \{\varnothing, \ell, h\}$ at rate $\rho e(x, y)$.[4]

[4]For simplicity, we assume that search on the match and out of the match are equally intensive. In equilibrium, payoffs while matched are strictly positive. Then, unmatched agents accept all

The set of partners an agent is willing to accept depends on her current match. A decision function $d\left(x,y,y'\right):\{\ell,h\}\times\{\ell,h\}\times\{\ell,h\}\to[0,1]$ specifies the probability that an agent of type x matched to an agent of type y would dissolve that match upon meeting a willing partner of type y'. The rate at which an agent of type x meets an agent of type y who is willing to form a match with her is denoted by $q(x,y):\{\ell,h\}\times\{\ell,h\}\to\mathbb{R}_+$ and given by $q(x,y)\equiv\rho\left[e\left(y,\varnothing\right)+\sum_{x'\in\{\ell,h\}}e\left(y,x'\right)d\left(y,x',x\right)\right]$.

Agents only get utility from flow payoffs. Flow payoffs are constant for the duration of the match, and are determined through bargaining, as discussed in the next subsection. Let $\pi\left(x,y\right):\{\ell,h\}\times\{\ell,h\}\to[0,f(x,y)]$, with $\pi(x,y)+\pi(y,x)\leq f(x,y)$, be the flow payoff agent x receives when matched to agent y. Unmatched agents obtain a zero flow payoff.[5]

We denote the value function of an x-type agent by $V(x,\varnothing)$ when she is unmatched and by $V(x,y)$ when she is matched to a y-type. Values are given by

$$\left[r+q\left(x,\ell\right)+q\left(x,h\right)\right]V(x,\varnothing)=0+q\left(x,\ell\right)V(x,\ell)+q\left(x,h\right)V(x,h)\quad\text{and}$$

$$\left(r+\delta+\sum_{y'\in\{\ell,h\}}d\left(x,y,y'\right)q\left(x,y'\right)+\sum_{x'\in\{\ell,h\}}d\left(y,x,x'\right)q\left(y,x'\right)\right)V(x,y)=\pi(x,y)$$
$$+\left(\delta+\sum_{x'\in\{\ell,h\}}d\left(y,x,x'\right)q\left(y,x'\right)\right)V(x,\varnothing)+\sum_{y'\in\{\ell,h\}}d\left(x,y,y'\right)q\left(x,y'\right)V\left(x,y'\right).$$

It is usually more convenient to work directly with the surplus agents obtain relative to being unmatched. Surplus $S\left(x,y\right):\{\ell,h\}\times\{\ell,h\}\to\mathbb{R}$ is given by

$$S\left(x,y\right)=\left(r+\delta+\sum_{x'\in\{\ell,h\}}d\left(y,x,x'\right)q\left(y,x'\right)\right)^{-1}\left[\pi(x,y)\right. \tag{1}$$
$$\left.+\sum_{y'\in\{\ell,h\}}d\left(x,y,y'\right)q\left(x,y'\right)\left[S\left(x,y'\right)-S(x,y)\right]-\sum_{y'\in\{\ell,h\}}q\left(x,y'\right)S\left(x,y'\right)\right].$$

partners. In Section 5.1 we allow search intensities to differ.

[5]Re-scaling the production function to allow for positive payoffs while unmatched leads to equivalent results.

We distinguish individual surpluses $S(x,y)$ and $S(y,x)$ from the total surplus of the match $S(x,y)+S(y,x)$ because the total surplus is not necessarily split symmetrically (as we show in the next subsection).

2.1 Timing and Bargaining

We propose the following timing. When two agents meet they observe each others' type. Before bargaining, each agent decides whether to create a match together. If both agents are willing to form a match, a transition occurs, and any previous match is dissolved. Therefore, when an agent bargains with her partner, she cannot exploit the existence of an alternative partner to improve her bargaining position. As a result, the outside option is always the value of being unmatched.[6]

When agents search on the match, the bargaining set is non-standard, so we need to describe it carefully. Once agents x and y form a match, they bargain on how to split the output. This allocation of output remains in place until the match breaks (exogenously or endogenously). Whenever a matched agent meets a potential partner with whom she anticipates a higher surplus, she leaves her current partner. Agents cannot commit not to leave each other, and cannot engage in renegotiation when an offer arrives.

The state of the economy is summarized by $S^* = \{S^*(x,y)\}_{(x,y)\in X^2}$, the surplus that agents obtain in each possible match and by $q^* = \{q^*(x,y)\}_{(x,y)\in X^2}$, the likelihood of finding willing partners. Agents take the state of the economy as given.

A possible agreement $c = \left(\widehat{d}, \widehat{\pi}\right)$ between x and y specifies both a decision function $\widehat{d}$ and an allocation $\widehat{\pi}$. Let $\widehat{d} = \left(\left\{\widehat{d}_1\left(y'\right)\right\}_{y'\in X}, \left\{\widehat{d}_2\left(x'\right)\right\}_{x'\in X}\right)$ and $\widehat{\pi} = (\widehat{\pi}_1, \widehat{\pi}_2)$, with $\widehat{\pi}_1 + \widehat{\pi}_2 \leq f(x,y)$. For example, $\widehat{d}_1\left(y'\right)$ denotes agent x's decision when faced with the possibility to match a (willing) agent of type y'. Taking the state of the economy (S^*, q^*) as given, an agreement $c = \left(\widehat{d}, \widehat{\pi}\right)$ induces surplus

[6]The timing of match-to-match transitions follows Pissarides [1994] and several recent papers (Shimer [2006], Gautier, Teulings, and Van Vuuren [2010], and Bartolucci [2013]). In Section 5.2 we allow agents to make counteroffers à la Kiyotaki and Lagos [2007]. Positive assortative matching can also arise in this case, even without complementarity in production.

pair $\widehat{S}^c = \left(\widehat{S}_1^c, \widehat{S}_2^c\right)$ with

$$\widehat{S}_1^c = \left(r + \delta + \sum_{x' \in X} \widehat{d}_2\left(x'\right) q^*\left(y, x'\right)\right)^{-1} \Big[\widehat{\pi}_1 + \sum_{y' \in X} \widehat{d}_1\left(y'\right) q^*\left(x, y'\right) \left[S^*\left(x, y'\right) - \widehat{S}_1^c\right] - \sum_{y' \in X} q^*\left(x, y'\right) S^*\left(x, y'\right)\Big],$$

and $\widehat{S}_2^c$ defined accordingly.

Since there is no renegotiation or commitment, only consistent agreements can occur:

Definition 1. Consistent Agreements. *Fix the state of the economy (S^*, q^*). An agreement $c = \left(\widehat{d}, \widehat{\pi}\right)$ is consistent if for all $y' \in X$,*

$$\widehat{d}_1\left(y'\right) \begin{cases} = 1 & \text{if } S^*(x, y') - \widehat{S}_1^c > 0 \\ \in [0, 1] & \text{if } S^*(x, y') - \widehat{S}_1^c = 0 \\ = 0 & \text{if } S^*(x, y') - \widehat{S}_1^c < 0 \end{cases}$$

and the same holds for $\widehat{d}_2\left(x'\right)$, for all $x' \in X$.

With this definition in hand, we can define our bargaining sets:

Definition 2. Bargaining Sets $\mathcal{S}$ under On-the-Match Search. *Fix the state of the economy (S^*, q^*). Agents x and y bargain over*

$$\mathcal{S}_{xy} = \left\{(S_1, S_2) : \exists \text{ consistent } c \text{ with } \widehat{S}_1^c = S_1 \text{ and } \widehat{S}_2^c = S_2\right\}.$$

Bargaining sets under on-the-match search have features that make the bargaining problem non-trivial. They may be non-convex, so Nash's assumptions [1950] are not satisfied.[7] Kaneko [1980] presents an extension of Nash's model that

[7] Bargaining sets may also be non-comprehensive. $\mathcal{S}$ is comprehensive if $0 \leq x \leq y$ and $y \in \mathcal{S}$ implies $x \in \mathcal{S}$. Non-comprehensiveness makes the analysis in Zhou [1997] and others unapplicable in a setup with on-the-match search. See Figure 1 in next section for an example of how bargaining sets look with on-the-match search.

allows for non-convex sets. Kaneko's version of Nash's axioms permits set-valued decision functions. A decision correspondence ϕ assigns to each compact subset S of $\mathbb{R}^2_+$ a non-empty subset $\phi(S) \subset S$.[8] Kaneko shows that a decision correspondence ϕ satisfies his axioms if and only if it maximizes the product of individual surpluses:

$$\phi(S) = \left\{ \left(\overline{S}_1, \overline{S}_2\right) \in S : \overline{S}_1 \overline{S}_2 \geq S_1 S_2 \ \text{ for all } \ (S_1, S_2) \in S \right\} \tag{2}$$

In our model, the bargaining sets $\mathcal{S}_{xy}$ are compact (see Appendix A.1 for details). From now on we assume that $\phi(\cdot)$ defined in (2) is the solution to the bargaining problem.

The solution to the bargaining problem maximizes the product of individual surpluses, as in Nash [1950]. However, the total surplus is not always split symmetrically because bargaining sets are non convex. To see why, consider an example with only two types of agents: $x, y \in \{\ell, h\}$, with ℓ slightly less than h. Assume agents produce $f(x, y) = x + y$ if matched and zero otherwise. If ℓ and h split the total surplus symmetrically, the low-type agent makes marginally more than ℓ per period but is dismissed when the high-type agent finds a high-type partner. Therefore, it is more convenient for the low-type agent to receive a per-period payoff of ℓ and get a larger expected duration of the match. The high type also benefits from that. Then, for $\ell \approx h$, the outcome from even surplus splitting is dominated.

2.2 Equilibrium

We can now formulate our notion of equilibrium in this economy.

Definition 3. Equilibrium with On-the-Match Search. *Take a pair of decision functions and allocations (d^*, π^*), its induced state of the economy (S^*, q^*) and*

[8] There are three main differences between Nash's and Kaneko's axioms. First, Kaneko assumes strict Pareto Optimality, whereas Nash assumes a weak version. Second, the axiom of independence of irrelevant alternatives (IIA) is now: $T \subset S$, $\phi(S) \cap T \neq \varnothing \Rightarrow \phi(T) = \phi(S) \cap T$. This is consistent with Nash's IIA, but it is a fairly restrictive version. Third, Kaneko assumes a weak form of continuity in the choice correspondence ϕ.

its resulting bargaining sets $\{\mathcal{S}_{xy}\}_{(x,y)\in X^2}$. We say that (d^*, π^*) is an equilibrium if for all $(x,y) \in X^2$,

1. *agreements are consistent,*[9]

2. *surpluses solve the bargaining problem: $(S^*(x,y), S^*(y,x)) \in \phi\left(\mathcal{S}_{xy}\right)$, and*

3. *market outcomes are robust: $S^*(x,y) > S^*(y,x) \Rightarrow \exists\ y' \neq y : S^*(x,y) = S^*(x,y')$.*

Before presenting our results, we provide a short discussion of our definition of equilibrium and its properties. First, equilibrium outcomes have some straightforward properties. For all matches, allocations exhaust production: $\pi(x,y) + \pi(y,x) = f(x,y)$. Moreover, agents only perform match-to-match transitions if they are strictly better off after the transition: $d\,(x,y,y') = \mathbb{1}\{S^*(x,y') > S^*(x,y)\}$. These results are direct consequences of the assumption of Strict Pareto Optimality in bargaining. Second, our model is symmetric in that both sides come from the same population. Thus, by construction, a low firm matched to a high worker obtains the same surplus as a low worker matched to a high firm. Third, we focus on equilibria where behavior is a function of own type and partner's type. As a result, equilibrium outcomes with two agents of the same type are symmetric.

Condition 3 in our definition of equilibrium is desirable, although not necessary for our message. We include it for two reasons. First, when it does not hold, the equilibrium does not survive a positive cost of match-to-match transition (we elaborate on this in Appendix A.2). In that sense, it is a robustness condition which restricts the set of equilibria. Our main insight is about existence of equilibria featuring positive assortative matching. It is prudent then to follow a conservative approach. Second, beyond robustness, condition 3 provides tractability to the model. It imposes symmetric surplus splitting in matches with strict preferences over partners' type.

[9]For each match $(x,y) \in X^2$, define agreement $c = \left(\widehat{d}, \widehat{\pi}\right)$ by $\widehat{d} = (d^*(x,y,y'), d^*(y,x,x'))$ and $\widehat{\pi} = (\pi^*(x,y), \pi^*(y,x))$.

2.3 Assortative Matching

An equilibrium decision function d^* induces a steady state distribution of matches $e(x, y)$. We argue that this steady state distribution can be positively assortative due to frictions.

In Becker's frictionless market [1973] there is positive assortative matching if agents only match with partners of their same type. In contrast, when it takes time to find a partner, agents may form matches with more than one type of partner; hence Shimer and Smith [2000] define sorting in terms of acceptance sets. However, if an agent can search while matched, her acceptance set depends not only on her own type, but also on her *current* partner's type. Since match-to-match transitions shape the steady state distribution, a characterization of the acceptance sets of unmatched agents is not enough to describe the sorting pattern. Therefore we use the following definition proposed by Lentz [2010] to describe sorting in markets with match-to-match transitions.

Definition 4. Positive Assortative Matching. *Take any $x_1, x_2 \in X$ with $x_1 > x_2$. There is positive assortative matching if and only if the distribution of partners of x_1 first order stochastically dominates the distribution of partners of x_2.*

3. Solution for the Two Type Case

The main insight of this paper is that frictions are a driving force towards positive sorting. Frictions generate rents, and rent splitting may induce an equilibrium preference for higher types. From now on, we say an equilibrium features *hyperphily* when $S^*(x, h) > S^*(x, \ell)$ for all $x \in \{\ell, h\}$. Frictions leads to sorting since hyperphily implies positive assortative matching.

Lemma 1. *In an equilibrium with hyperphily and two types, h's distribution of partners first order stochastically dominates ℓ's.*

See Appendix A.5 for the proof.

We now present necessary and sufficient conditions for the existence of an equilibrium with hyperphily. Then, we present a complete characterization of the model. We describe all possible equilibria and the conditions for their existence. This allows us to state necessary and sufficient conditions for hyperphily to be the unique equilibrium.

3.1 An Equilibrium with Hyperphily

Under hyperphily, since no agent is indifferent between partners of different types, the total surplus of the match is split evenly in all matches (see equilibrium definition). Therefore, the equilibrium allocations are given by $\pi^*(\ell, \ell) = \ell$, $\pi^*(h, h) = h$, and $\pi^*(\ell, h)$ is set so that $S^*(\ell, h) = S^*(h, \ell)$.

As explained in the previous section, agents' transitions must be consistent with the surplus they obtain in each match. Moreover, we require that, for each match, no consistent agreement leads to a higher product of individual surpluses. Thus, the agreement between agents must be a global maximum in the bargaining set. This is a restrictive condition, which is not easy to check in general. We check each match step by step.

Pair (d^*, π^*) is consistent in an equilibrium with hyperphily if the resulting surpluses satisfy

$$S^*(h, h) > S^*(h, \ell) \quad \text{and} \quad S^*(\ell, h) > S^*(\ell, \ell)\,. \tag{3}$$

We discuss next when (d^*, π^*) solves the bargaining problem for each possible match.

Bargaining Solution in Match (ℓ, h)

Total surplus is split evenly between ℓ and h. An agreement leading to a higher product of individual surpluses can only exist if it also induces a larger *total* surplus. Since ℓ does not leave the match (ℓ, h) under hyperphily, a larger total surplus can only be reached in the match (ℓ, h) if h chooses not to leave. Thus, we study consistent agreements between ℓ and h where h does not leave. Let $\left(\widehat{S}^c_\ell, \widehat{S}^c_h\right)$ denote the surplus in some alternative agreement c. h does not leave for a high-type agent only if $\widehat{S}^c_h \geq S^*(h,h)$.

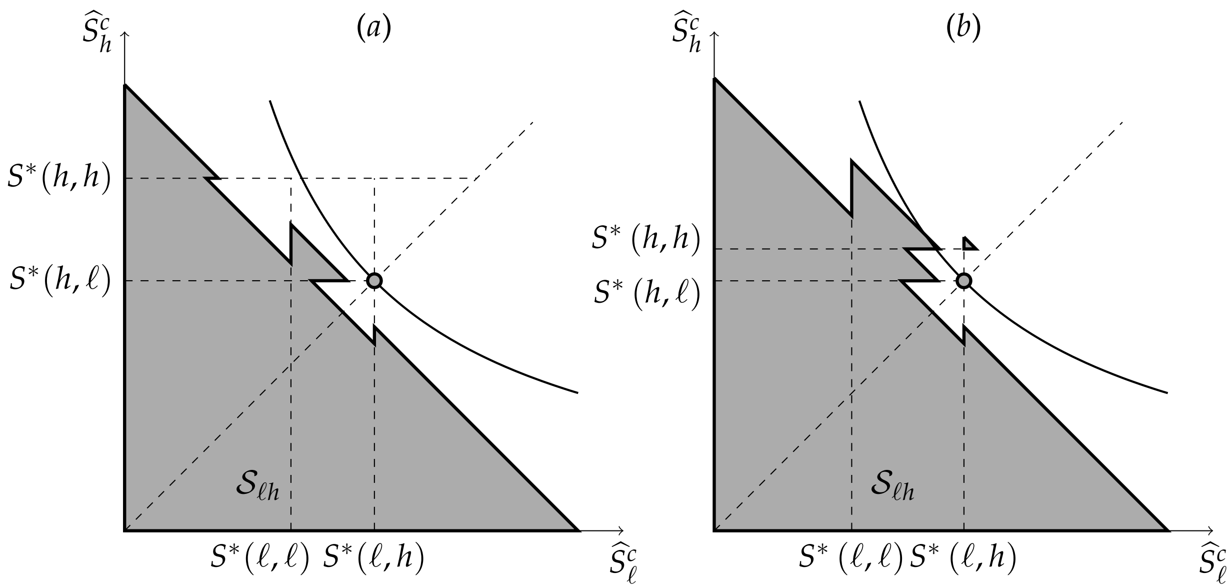
There are three possible kinds of agreements with h staying. In the first kind of agreement (c_1), both ℓ and h choose not to leave each other. In the second one (c_2), h always stays, but ℓ leaves when she finds a new h. In the third one (c_3), h always stays, but ℓ leaves when she finds *any* new partner. If the first kind of agreement exists, it makes both agents better off, so our original candidate is not an equilibrium. The second and third cases involve ℓ obtaining a lower surplus. We need to check whether a higher product of individual surpluses is attained in these cases. Pair (d^*, π^*) solves the bargaining problem in match (ℓ, h) if and only if Condition 1 holds.

CONDITION 1. *Let c_1, c_2 and c_3 be defined as stated. No allocation generates*

$$\begin{aligned}
&\widehat{S}^{c_1}_h \geq S^*(h,h) \text{ and } && \widehat{S}^{c_1}_\ell \geq S^*(\ell,h), \text{ or} && \\
&\widehat{S}^{c_2}_h \geq S^*(h,h), && S^*(\ell,\ell) \leq \widehat{S}^{c_2}_\ell < S^*(\ell,h) && \text{and } \widehat{S}^{c_2}_\ell \widehat{S}^{c_2}_h > S^*(\ell,h) S^*(h,\ell), \text{ or} \\
&\widehat{S}^{c_3}_h \geq S^*(h,h), && \widehat{S}^{c_3}_\ell < S^*(\ell,\ell) && \text{and } \widehat{S}^{c_3}_\ell \widehat{S}^{c_3}_h > S^*(\ell,h) S^*(h,\ell).
\end{aligned}$$

Figure 1 presents two examples to illustrate how bargaining sets are built and how to verify Condition 1. As mentioned in the previous section, the trade-off between expected duration and flow payoff makes the bargaining sets non-convex. To see why, take the boundary of bargaining set $\mathcal{S}_{\ell h}$ in panel *a* in Figure 1. Consider first the point that gives ℓ zero surplus and h his maximum possible surplus on $\mathcal{S}_{\ell h}$. At this point, h never leaves the match, while ℓ gets $\pi(\ell, h) = 0$ so she leaves for any alternative partner (of either type). An increase in $\pi(\ell, h)$, together with its corresponding decrease in $\pi(h, \ell)$, increases $\widehat{S}^c_\ell$ in the same amount as $\widehat{S}^c_h$ decreases. Thus, for small changes in flow payoffs, the boundary of the bargaining set is linear. However, consider now the point where $\widehat{S}^c_h = S^*(h,h)$. A further increase in $\pi(\ell, h)$ makes $\widehat{S}^c_h < S^*(h,h)$, so h starts leaving whenever she finds another h. The expected duration of the match decreases, and although $\pi(\ell, h)$ is higher, $\widehat{S}_\ell^c$ decreases discretely. It is this jump that generates a non-convexity in the bargaining set. In general, bargaining sets are non-convex in the neighborhood of agreements leading to indifference.

Figure 1: Bargaining Sets $\mathcal{S}_{\ell h}$



Note: $\rho = 0.1$, $r = 0.1, \delta = 0.05, \ell = 1$ and $h = 2$. In (a), $F = \ell + h$. In (b), $F = 1.6\ell + h$.

As Figure 1 illustrates, bargaining sets are built from potentially disjoint compact sets. In fact, agreement (d^*, π^*) maps to an isolated point in the bargaining set. Any marginal deviation from π^* decreases the expected duration of the match discretely. This occurs because the partner whose flow payoff has been reduced now leaves when she finds a new partner of the *same* type as her current one.

Panel *a* shows a case where condition 1 holds: hyperphily solves the bargaining problem in the match (ℓ, h). The shaded area in panel *a* represents the bargaining set $\mathcal{S}_{\ell h}$ under hyperphily and a modular production function. The curve through $(S^*(\ell,h), S^*(h,\ell))$ indicates all points attaining product $S^*(\ell,h) \times S^*(h,\ell)$. No element in the bargaining set attains a higher product of individual surpluses. Note this occurs without complementarity in production and with patient agents.

Panel *b* shows a case where condition 1 does not hold. When the production function is sufficiently submodular hyperphily is no longer an equilibrium. An alternative consistent agreement leads to a higher product of individual surpluses *and* to a higher individual surplus for both agents. Agent ℓ receives *less* than half of a *larger* surplus in order to make her partner indifferent. Still, agent ℓ is better off. This violates the first line of Condition 1.

In the example presented in panel b the second line of Condition 1 is also violated. An agreement that makes 1) h indifferent to a match with another h and 2) ℓ worse off than in a match to a different h is also consistent and leads to a larger product of individual surpluses.

Bargaining Solution in Match (ℓ, ℓ)

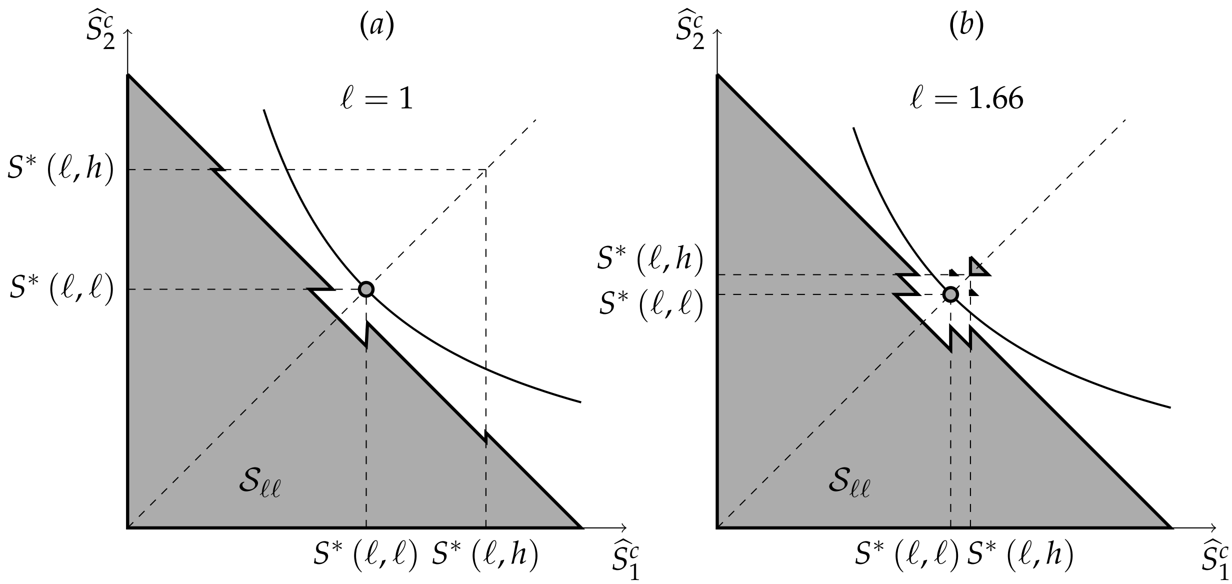
As in match (ℓ, h), there are three cases to consider. In the first (c_4), both agents choose not to leave each other. In the second (c_5), one ℓ agent never leaves while the second one leaves only when finding a willing h. In the third (c_6), one ℓ agent never leaves while the other one leaves when finding *any* willing partner. Let $\left(\widehat{S}_1^c, \widehat{S}_2^c\right)$ denote the surplus in an alternative contract c. Pair (d^*, π^*) solves the bargaining problem in match (ℓ, ℓ) if and only if Condition 2 holds.

CONDITION 2. *Let c_4, c_5 and c_6 be defined as stated. No allocation generates*

$$\begin{array}{lll} \widehat{S}_1^{c_4} \geq S^*(\ell, h), \text{ or} & & \\ \widehat{S}_1^{c_5} \geq S^*(\ell, h), \text{ and} & S^*(\ell, \ell) \leq \widehat{S}_2^{c_5} < S^*(\ell, h), \text{ or} & \\ \widehat{S}_1^{c_6} \geq S^*(\ell, h), & \widehat{S}_2^{c_6} < S^*(\ell, \ell) \text{ and} & \widehat{S}_1^{c_6}\widehat{S}_2^{c_6} > \left[S^*(\ell, \ell)\right]^2. \end{array}$$

We present again two examples to illustrate bargaining, this time on match (ℓ, ℓ). Panels a and b in Figure 2 present bargaining set $\mathcal{S}_{\ell\ell}$ with hyperphily and a modular production function. In panel b, types are closer: $\ell = 1.66$ and $h = 2$, whereas in panel a, $\ell = 1$ and $h = 2$. It is easy to see that hyperphily solves the bargaining problem in panel a. In panel b, however, an alternative agreement with both ℓ agents choosing not to leave each other makes them better off, so hyperphily does not solve the bargaining problem.

Figure 2: Bargaining Sets $\mathcal{S}_{\ell\ell}$



Note: $\rho = 0.1$, $r = 0.1, \delta = 0.05, h = 2$ and $F = \ell + h$.

In the example presented in panel b, the second line in Condition 2 is also violated. An agreement that makes 1) one ℓ indifferent to a match with h and 2) the second ℓ at least as well off as before is also feasible.

Bargaining Solution in Match (h, h)

There is no endogenous destruction in match (h, h) and agents split the surplus evenly. Therefore, no consistent alternative agreement leads to a higher product of individual surpluses.

Equilibrium with Hyperphily

Our first proposition summarizes the necessary and sufficient conditions for hyperphily.

PROPOSITION 1. EQUILIBRIUM WITH HYPERPHILY. *Conditions 1 and 2 are necessary and sufficient for hyperphily. We solve them explicitly and characterize the set of* *primitives* $(\ell, h, F, r, \rho, \delta)$ *such that an equilibrium with hyperphily exists.*

Proof. Equation (3), and Conditions 1 and 2 generate 8 inequalities which determine when hyperphily can be an equilibrium. Whenever Conditions 1 and 2 are satisfied, then equation (3) also is. We express Conditions 1 and 2 as explicit functions of $(\ell, h, F, r, \rho, \delta)$. We present the details in Appendix A.6. ■

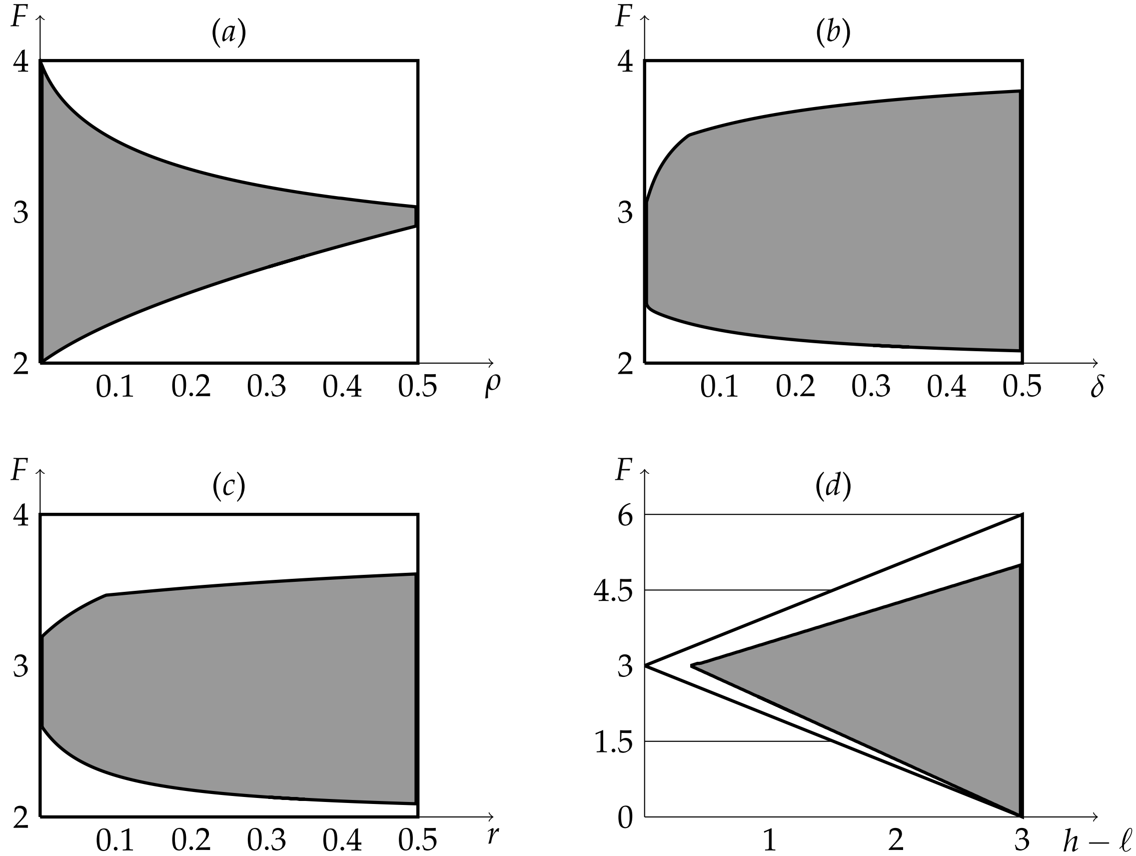
Figure 3 illustrates the set of primitives $(\ell, h, F, r, \rho, \delta)$ which lead to hyperphily. The shaded areas in panels a, b, c and d represent the set of values of F consistent with an equilibrium with hyperphily as a function of the matching rate ρ, the destruction rate δ, the discount rate r and the difference between $h - \ell$ respectively.

As we see in panel a, low values of ρ allow for hyperphily even when the production function is significantly submodular. As ρ decreases, the probability that h leaves the match (ℓ, h) becomes lower, so compensating her to make her stay becomes less attractive. In the limit as $\rho \to 0$, hyperphily is an equilibrium for all degrees of complementarity in the production function. On the other side, as $\rho \to \infty$, the duration of any match with voluntary destruction approaches zero. Thus, hyperphily cannot be an equilibrium.

As we see in panel b, higher values for the destruction rate δ make hyperphily more likely. As δ increases, endogenous destruction becomes less relevant relative to exogenous destruction. Therefore the maximum degree of submodularity which supports hyperphily increases. As $\delta \to \infty$, the duration of every match goes to zero independently of the allocation of production, so hyperphily holds for every value of the other primitives. On the other side, lower values of δ leave less room for hyperphily. When δ is low, there are few unmatched agents. Being unmatched becomes relatively less attractive, since it takes a long time to find a partner. However, if agents are impatient enough, as $\delta \to 0$ there are still equilibria with hyperphily, even when the production function is submodular.

Panel c illustrates the intuition discussed in the Introduction. As agents become more impatient (higher r), complementarity in production becomes less important relative to rent splitting. In the limit as $r \to \infty$, hyperphily is an equilibrium for

Figure 3: Existence of Equilibrium with Hyperphily



Note: In (a), $\ell = 1, h = 2, \delta = 0.05$ and $r = 0.1$. In (b), $\ell = 1, h = 2, \rho = 0.1$ and $r = 0.1$. In (c), $\ell = 1, h = 2, \delta = 0.05$ and $\rho = 0.1$. In (d), $r = 0.1, \rho = 0.1, \delta = 0.05$ and $\ell + h = 3$, with $0 < \ell < 1.5 < h < 3$.

any degree of complementarity in the production function. When agents are patient, there are equilibria with hyperphily provided that the complementarity in production is not too strong.

Panel d illustrates the example discussed in Section 2.1. When the difference between types is close to zero, ℓ does not get much from extracting surplus from h. Thus, ℓ makes h indifferent, so he does not leave for another h. Agreement c_1 leads to a higher product of surpluses in match (ℓ, h). As $h - \ell$ increases, hyperphily becomes an equilibrium for a larger range of values of F. Moreover, as ℓ approaches 0, hyperphily holds even for a significantly submodular production function.

3.2 All Possible Equilibria

We present a complete characterization of equilibria in this subsection. Depending on the value of the primitives, several different equilibria arise in our simple two type model. In principle, there could be nine different types of equilibria, each associated to a different vector d^*. Table 1 shows all of them. We present necessary and sufficient conditions for the existence of all types of equilibrium. Thus, we obtain necessary and sufficient conditions for hyperphily to be the only possible equilibrium.

Positive assortative matching can arise not only with hyperphily but also with strict or weak homophily.[10] Therefore, a full characterization of the model allows us to present necessary and sufficient conditions for the existence and uniqueness of an equilibrium with positive assortative matching.

Characterizing each equilibrium involves going through the same process as already performed for hyperphily. First, we select agreements that satisfy condition 3 in our equilibrium definition. Then, we verify that transitions are consistent. Lastly, for each possible match, we verify that the equilibrium agreement solves the bargaining problem.

PROPOSITION 2. ALL EQUILIBRIA WITH TWO TYPES. *For each possible type of equilibrium in Table 1 we characterize explicitly the set $(\ell, h, F, r, \rho, \delta)$ such that the equilibrium exists.*[11]

We discuss now the main results regarding other equilibria. First, note that with a supermodular production function the equilibrium cannot feature neither weak nor strict heterophily. To see this, note that $\pi^*(h,\ell) < h$ makes a h-type agent strictly prefer another agent of type h. Similarly, $\pi^*(\ell,h) < \ell$ makes an ℓ-type agent strictly prefer another agent of type ℓ. Then, $F \geq h + \ell$ is a necessary

[10] In the Online Appendix, for each possible equilibrium, we present closed-form solutions for densities $e(x,y)$ and we show the sorting pattern that arises. In Table 1 we indicate if sorting is positive, negative, or random. Equilibria with weak heterophily and weak homophily (B) feature no first order stochastic dominance.

[11] The sets are obtained analogously to those from Proposition 1. We present closed-form solutions for these sets (and how they are obtained) in the Online Appendix.

Table 1: All Possible Equilibria in the Two Type Model

ℓ's decision	h's decision		
	$d^*(h,\ell,h)=1$ $d^*(h,h,\ell)=0$	$d^*(h,\ell,h)=0$ $d^*(h,h,\ell)=0$	$d^*(h,\ell,h)=0$ $d^*(h,h,\ell)=1$
$d^*(\ell,\ell,h)=1$ $d^*(\ell,h,\ell)=0$	Hyperphily (positive sorting)	Weak Heterophily	Strict Heterophily (negative sorting)
$d^*(\ell,\ell,h)=0$ $d^*(\ell,h,\ell)=0$	Weak Homophily (A) (positive sorting)	Indifference (random sorting)	*Impossible*
$d^*(\ell,\ell,h)=0$ $d^*(\ell,h,\ell)=1$	Strict Homophily (positive sorting)	Weak Homophily (B)	*Impossible*

condition for both weak and strict heterophily. It is also straightforward to show that neither weak nor strict homophily can be equilibria with a submodular production function. Finally, only strict heterophily exists when h strictly prefers ℓ to h (see Appendix A.3 for details).

Strict heterophily is the only equilibrium featuring negative assortative matching. Thus, negative sorting only occurs with a submodular production function. Positive assortative matching occurs both with homophily and hyperphily. Random sorting only happens if both h and ℓ are indifferent, which requires $\pi^*(\ell,h)=\ell$ and $\pi^*(h,\ell)=h$. Hence indifference, and therefore random sorting, can only happen if the production function is modular.

Figure 4 illustrates the set of primitives which lead to each possible equilibrium.[12] Equilibria with strict heterophily or strict homophily are rare, as shown in panel *c* of Figure 4. In strict heterophily h prefers a match with ℓ over a more productive match with another h. This can happen when ℓ's outside option is lower than h's. Therefore, although the production of the match (ℓ,h) is smaller than the production of the match (h,h), the total surplus of the match (ℓ,h) is larger than the total surplus of the match (h,h). On the other hand, strict homophily requires ℓ to strictly prefer another ℓ, which is demanding given that the match (ℓ,h) is more productive. As in the case of strict heterophily, the agent prefers a less productive

[12]The shaded areas in Figure 4 represent the set of values of F consistent with each equilibrium as a function of the destruction rate δ. In the Online Appendix we present the corresponding figures for ρ, r, and $h-\ell$.

match because its total surplus is larger. When r or δ increase, or when ρ decreases, the outside option becomes less relevant and therefore strict homophily and strict heterophily require stronger complementarity in production.

If agents can search while matched, the match duration depends on the bargaining outcome. Symmetric surplus splitting might not solve the bargaining problem, as it occurs in the cases of weak heterophily and weak homophily. In these equilibria, one agent is indifferent between partner types and takes a larger fraction of the total surplus in the match (ℓ, h). Uneven surplus splitting produces a larger product of surpluses because it implies a longer duration of the match and a larger total surplus. These equilibria are more likely to exist when agents care more about endogenous destruction (when r or δ are low); or when it is easier to find partners (when ρ is large). This is shown in panel b and d of Figure 4.

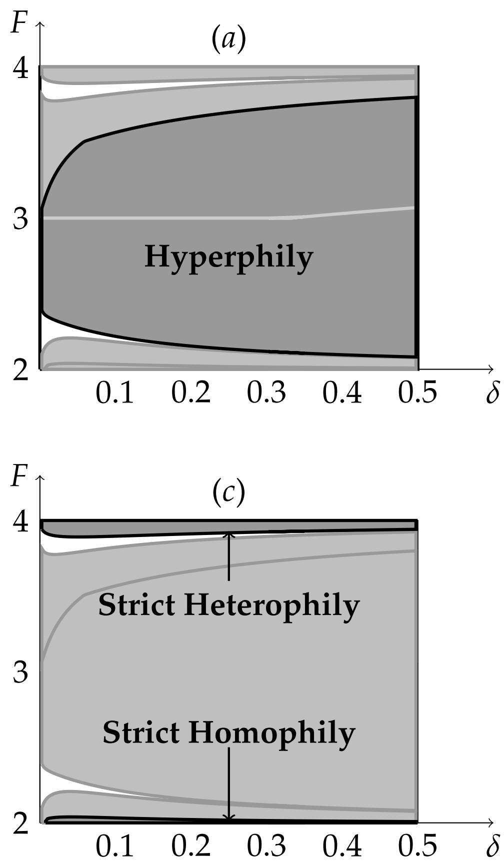
As frictions vanish, the outcome does not necessarily approach that of the frictionless market in Becker [1973]. Consider the index of labor market frictions $\kappa \equiv \frac{\rho}{\delta}$.[13] A larger κ implies weaker frictions. With submodularity, one would expect perfect negative sorting in a frictionless market. In contrast, we show that hyperphily, and thus positive sorting, can arise with submodularity when $\kappa \to \infty$. The equilibrium outcome in the limit depends on whether it is ρ or δ what drives $\kappa \to \infty$. On one side, if κ is large because ρ is large, hyperphily does not occur. On the other side, if κ is large because δ is small, there are equilibria with hyperphily, even with a submodular production function. Patient enough ℓ-type agents are happy to trade a shorter duration of the match for a higher allocation. Interestingly, as $\delta \to 0$ sorting becomes *perfectly* positive, instead of perfectly negative as in Becker [1973].

As frictions grow, positive assortative matching becomes pervasive. When δ and r increase, or when ρ decreases, the region where hyperphily is the unique equilibrium grows. In the next section we obtain this as a general result for any number of types. This result goes against the idea that stronger frictions require

[13] κ is used as an index of frictions in several papers. See Ridder and van den Berg [2003] for an example.

stronger complementarity in production for the equilibrium to be positively assortative.

Figure 4: The Impact of Destruction Rate δ



Note: $\ell = 1, h = 2, r = 0.1$ and $\rho = 0.1$.

4. The Case with N Types

We extend now the intuition described in the introduction and developed for two types to the case with any finite number of types. With a large number of types one cannot characterize equilibrium behavior for all parameter values. We study the case with N types in two ways. First, we consider the case with impatient agents (high values of r), high exogenous destruction rates (high values of δ), or low meeting rates (low values of ρ). We show that for low enough $\frac{\rho}{r+\delta}$, hyperphily is an equilibrium, and no other equilibrium exists.[14] Second, we present numerical examples.

We extend the model to allow for N types: $x \in X = \{1, 2, \ldots, N\}$. Types are in equal proportion in the population: $e(x, y) : X \times X \cup \{\varnothing\} \to \left[0, \frac{1}{N}\right]$ has $\sum_{y \in X \cup \{\varnothing\}} e(x, y) = \frac{1}{N}$. All other functions are modified appropriately to allow for N types.

4.1 Sufficient Conditions for Hyperphily with N Types

Hyperphily is the unique equilibrium when agents become impatient, or when endogenous destruction becomes less relevant. When $\frac{\rho}{r+\delta}$ is low, continuation values become less relevant. Therefore, individual surpluses depend mostly on current payoffs, which are close to an equal split of output. Payoffs then depend on total output, which increases in the partner's type. As a result, surplus is increasing in the partner's type for low values of $\frac{\rho}{r+\delta}$. Then, no equilibrium other than hyperphily can exist. Additionally, we show that hyperphily is an equilibrium. In that respect, note first that consistency is straightforward for small $\frac{\rho}{r+\delta}$, given our previous argument. Second, we show in Appendix A.4 how no alternative consistent agreement leads to a higher product of individual surpluses. Proposition 3 summarizes these findings.

PROPOSITION 3. HYPERPHILY WITH N TYPES. *When $\frac{\rho}{r+\delta}$ is low enough, (d^*, π^*) is an equilibrium if and only if it features hyperphily.*

See Appendix A.4 for the proof.

4.2 Numerical Examples with N Types

We provide now numerical examples in a model with N types for parameter values in line with the literature. We show that equilibria with hyperphily and positive

[14] We computed the steady state distribution under hyperphily for 1,000 values of $\kappa^{-1} \in (0, 1)$ and 1,000 values of $\kappa \in (0, 1)$ for $N = 10, 20$ and 100. In all cases there is positive assortative matching in the steady state distribution.

assortative matching arise without complementarity in production.

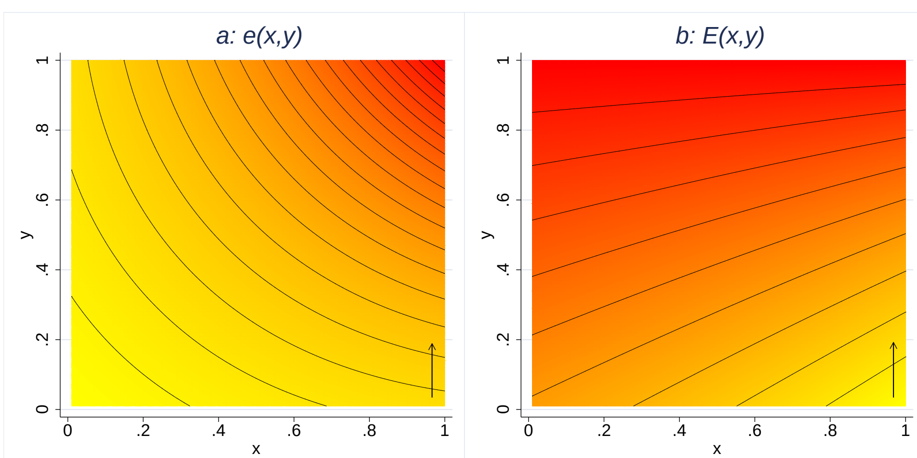
We solve the model by a nested fixed point algorithm. We start from a flat distribution of matches and calculate value functions for all possible matches. These first value functions induce preferences over partner types which we use to update the steady state distribution of matches. With the updated steady state distribution, we update the value functions. We iterate this process until we find a fixed point for both the steady state distribution of matches and the value functions.

We search specifically for equilibria without indifference over partners. When no agent is indifferent, symmetric surplus splitting solves the bargaining problem in all matches. Once we find a candidate set of value functions and distribution of matches that solves the model, we check that the solution maximizes the product of surpluses in all matches. To do this, for each match we evaluate all possible consistent agreements $c = \left(\widehat{d}, \widehat{\pi}\right)$, given our candidate. Our candidate solves the model if it maximizes the product of surpluses in every match.

Example 1 presents a case featuring hyperphily with $N = 100$.

EXAMPLE 1. *Types are uniformly distributed in a* 100*-point grid between* 0 *and* 1*. Production is modular:* $f(x, y) = x + y$*.* ($\delta = 0.05$, $r = 0.1$ *and* $\rho = 0.1$).

Figure 5: Positive Assortative Matching in Example 1



Note: Panel *a* presents the density of matches $e(x, y)$. Panel *b* presents the cumulative distribution of type x's partners $E(x, y)$. Darker points correspond to higher values.

Panel a in Figure 5 shows the probability distribution function $e(x, y)$ and panel b shows the cumulative distribution of type x's partners $E(x, y) \equiv \sum_{\tilde{y} \in X \cup \{\varnothing\}, \tilde{y} < y} e(x, \tilde{y})$. Sorting is defined in terms of stochastic dominance, so panel b is informative on sorting patterns. The strictly increasing contour lines of $E(x, y)$ show that the cumulative distribution of type x's is decreasing in own type, which implies positive assortative matching.

A similar result holds with a slightly submodular production function. However, if either δ or r decrease enough or if ρ increases enough, symmetric surplus splitting does not maximize the product of surpluses in some matches. Take Example 1 and double the search intensity (so $\rho = 0.2$). Now, hyperphily does not maximize the product of individual surpluses in matches where $|x - y|$ is large. For a given set of parameters, if there is at least one match where the agreement from hyperphily does not solve the bargaining problem, then hyperphily is not an equilibrium. However, this does not imply that matching is not positively assortative. As in the case with two types with weak and strict homophily, there may be other equilibria with positive assortative matching.

5. Extensions and Discussion

5.1 Different Search Intensities

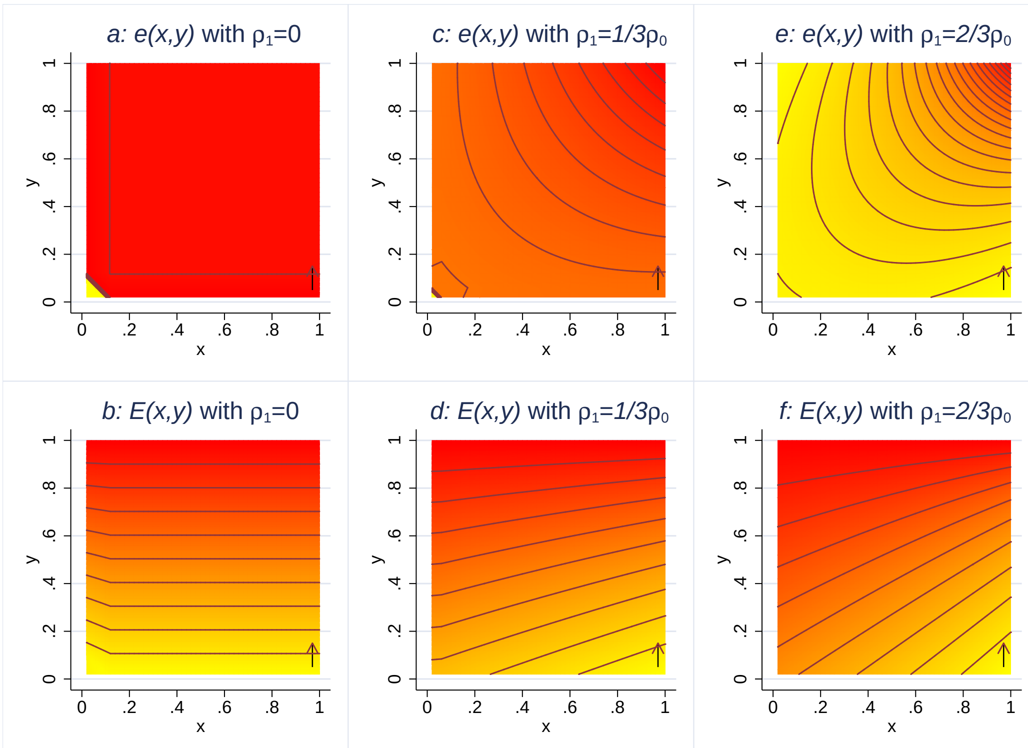
We now relax the assumption that on-the-match search efficiency is equal to search efficiency out of the match. Let ρ_0 denote the search intensity of an unmatched agent and let ρ_1 be the search efficiency of a matched one. The meeting rate is simply the product of the search intensities of those who meet.[15] The following example illustrates how different values of ρ_1 affect the equilibrium.

Example 2. *Types are uniformly distributed in a* 100*-point grid between* 0 *and* 1*. Pro-*

[15] For example, there are $\rho_0 \rho_1 e(\ell, \varnothing) e(h, h)$ unmatched ℓ-type agents who meet h-type agents matched to other h-type agents. Similarly, there are $(\rho_0)^2 e(\ell, \varnothing) e(h, \varnothing)$ unmatched ℓ-type agents who meet unmatched h-type agents. Our approach here is similar to Bobbio [2009].

duction is modular: $f(x,y) = x + y$. *(*$\delta = 0.05, r = 0.1$ *and* $\rho_0 = \sqrt{0.1}$*). Consider three cases:* $(i) : \rho_1 = 0, (ii) : \rho_1 = \frac{1}{3}\rho_0$, *and* $(iii) : \rho_1 = \frac{2}{3}\rho_0$.

Figure 6: Different On-the-Match and Out-of-the-Match Search Intensity in Example 2



Note: Panels a, c and e present the density of matches $e(x,y)$ for different values of ρ_1. Panels b, d and f present the cumulative distribution of type x's partners $E(x,y)$ for different values of ρ_1. Darker points correspond to higher values.

Positive sorting does not hold for low values of ρ_1, but it does as ρ_1 increases. Upper panels in Figure 6 show densities $e(x,y)$ while lower panels show cumulative distribution of type x's partners $E(x,y)$. Without on-the-match search $(\rho_1 = 0)$ the equilibrium features hyperphily and *negative* assortative matching: the contour lines of $E(x,y)$ are decreasing in x in panel b. When we allow agents to search on the match but with low search efficiency, there is assortative matching for agents of high type, but low-type agents still prefer to wait unmatched for more profitable partners. Therefore matching is not positively assortative for low type agents. With the parameter values used in these simulations, for values of search efficiency on the match as low as two thirds of the search efficiency out of the match, there is no difference in acceptance sets between unmatched agents of different types. All unmatched agents accept all partners, and since agents search on the match and prefer better partners, there is positive assortative matching in the whole support of types.

Example 2 also illustrates that hyperphily does not imply positive sorting when on-the-match search is not allowed. This is true in general in a model like Shimer and Smith [2000]. Since only unmatched agents search, the steady state distribution of types is shaped by acceptance sets. In an equilibrium with hyperphily, the total surplus of the match is strictly increasing in types. In this way, if a partner of type x is accepted by an agent of type y, then every $x' \geq x$ is also accepted by every $y' \geq y$. Therefore, the lower bound of the acceptance set is non increasing in own type (and there is no upper bound for hyperphily). As a result, when all agents accept everybody there is random sorting. Otherwise, there is negative sorting.

There is no positive sorting in Shimer and Smith [2000] with a modular, or slightly supermodular production function. It is easy to show from equations (5) and (8) in Shimer and Smith [2000] that if complementarity in production is weak enough the equilibrium features hyperphily. Hyperphily does not lead to positive assortative matching when agents are not allowed to replace their partners. Then, a slightly supermodular production function does not lead to positive assortative matching in their case.

5.2 An Example with Renegotiation

The main result in this paper is that frictions can lead to positive sorting, even without productive complementarity. In our stylized model, agents are not allowed to renegotiate how to split production when one of them meets an alternative partner. However, the mechanism we highlight can also hold if agents are allowed to renegotiate. We present next an example with on-the-match search, renegotiation and no complementarity in production that features hyperphily and positive assortative matching.

Since both partners' outside options change, modeling renegotiation with bilateral on-the-match search is not straightforward. Kiyotaki and Lagos [2007] present a search model where both the firm and the worker search on the match. In their setting, contracts can be renegotiated if a partner has a credible threat to dissolve the match. When an agent finds an alternative partner, her current partner and the poaching one compete *à la* Bertrand. Kiyotaki and Lagos do not study sorting since agents are homogeneous in their model. Matches are heterogeneous only due to a fixed match-specific productivity shock.

For simplicity, consider infinitely impatient agents of one of two types who bargain *à la* Kiyotaki and Lagos. As discussed in Section 4.1, in the limit the value of the match depends only on the flow-payoff received, and outside options converge to zero. Whenever an unmatched agent meets a matched one, both competing agents (the poaching one, and the current partner) have being unmatched as their outside option (with zero value). The more productive one can make a better offer, so he always wins. However, when matched agents meet other matched agents, some of the transitions that occur without renegotiation no longer happen.

Renegotiation prevents inefficient separations: the sum of the surplus of the destroyed matches cannot exceed the surplus of the newly created one. Therefore, in this simple example, when an h-type agent matched to an ℓ-type meets another h-type also matched to an ℓ-type, both h-type agents renegotiate their contracts and no match is destroyed (see Proposition 1 in Kiyotaki and Lagos). However, both h and ℓ still leave ℓ when they find an unmatched h. Therefore the steady-state distribution of partners of h first order stochastically dominates the distribution of partners of ℓ, as we show in Appendix A.5.1. In this way, although no inefficient separations take place when renegotiation is allowed, frictions can lead to positive assortative matching without productive complementarity.

5.3 The Planner's Problem

In this last section we look into which transitions maximize the output in steady state. We consider only two types of agents: $X = \{\ell, h\}$ and a modular production function. The decentralized equilibrium in this case features either positive sorting or random sorting.

The solution to the planner's problem features positive assortative matching when the planner has full control over transitions. Assume first that he can take into account current partners' types when deciding whether two agents who meet should form a match. An unmatched h-type is more costly than an unmatched ℓ-type. Therefore, when an agent matched to a partner of type ℓ meets an unmatched agent of type h, she replaces her current partner. When two matched agents meet, they do not form a new match since its output does not exceed the sum of the outputs of the destroyed matches. Then the planner chooses transitions as in Section 5.2 to maximize the economy's output.

The optimal decision of a planner restricted to choose decision functions d instead of transitions depends on primitives. When the difference $h - \ell$ is small enough, the planner has only unmatched agents forming a match. Otherwise, decision functions as in hyperphily are optimal. Then, the planner's solution features either positive sorting or random sorting in this case.

6. Conclusion

In this paper we show how frictions lead to positive assortative matching. While in frictionless markets payoffs reflect individual contributions, the division of output becomes more even when it takes time to find a partner. The total production of the match becomes the main determinant of preferences over partners when frictions are large. Production increases in partner's type, so an endogenous preference for better types (hyperphily) arises. When individuals search while matched, more productive agents upgrade their partners faster. The steady state distribution thus becomes positively assortative.

A key element in our analysis is that agents are allowed to search while matched. Match-to-match transitions are pervasive in markets with two-sided heterogeneity. We present a partnership model that includes the key elements of Shimer and Smith [2000] and allows for bilateral on-the-match search.[16] We first analyze the case where agents are of one of two types: either low or high productivity. We provide necessary and sufficient conditions for hyperphily and show that this preference leads to positive assortative matching. We highlight conditions such that positive assortative matching arises even with a modular or submodular production function. Our results extend to the case with any finite number of types. We show that as agents become impatient or frictions large, hyperphily is the unique equilibrium.

The conventional wisdom states that stronger frictions require stronger complementarity in production for positive assortative matching to arise. The intuition behind this view is straightforward: with frictions, agents only wait for their preferred partners if the complementarity is strong enough to compensate for the waiting cost. Our paper highlights a different role for frictions. Frictions modify the division of output and therefore shape preferences over partners. If frictions are strong, agents prefer higher types. Therefore, frictions can lead to positive sorting. Our result challenges the interpretation of sorting as evidence of complementarity in production in markets with frictions.

There are legal constraints on replacing partners in several markets with two-sided heterogeneity. Our results contribute to the discussion on the effects of match protection. First, we show that the planner uses match-to-match transitions in order to maximize the economy's output. Second, our results highlight the role of match-to-match transitions as a potential tool to mend the hold-up problem produced by frictions. When differences in types are a result of ex-ante investment

[16]Most recent studies on assortative matching in markets with frictions and transferable utility take the canonical model of Shimer and Smith [2000] as a starting point (see Lopes de Melo [2013], Hagedorn, Law, and Manovskii [2012], and Lise, Meghir, and Robin [2013]).

decisions, frictions reduce agents' incentives to invest (see for example Acemoglu and Shimer [1999b] or Flinn and Mullins [2014]). However, if agents are allowed to replace their partners, higher types agents can sort to avoid being held up. Then, under positive sorting, there are higher incentives to invest. A rigorous analysis of this point is beyond the scope of this paper, but we believe it deserves further investigation.

A. Appendix

A.1 Bargaining Sets are Compact

LEMMA 2. *Take any state of the economy (S^*, q^*). Then, bargaining sets $\mathcal{S}_{xy}$ under on-the-match search are compact.*

Proof. Since $r > 0$ and $f(x,y)$ is finite, $\mathcal{S}_{xy}$ is bounded. We show next that $\mathcal{S}_{xy}$ is also closed. Take a sequence $\left\{\left(S_1^n, S_2^n\right)\right\}_{n=1}^{\infty} \in \mathcal{S}_{xy}$ generated by a sequence of consistent agreements $\left\{\left(\widehat{d}^n, \widehat{\pi}^n\right)\right\}_{n=1}^{\infty}$ and with $\lim_{n\to\infty}\left(S_1^n, S_2^n\right) = \left(\overline{S}_1, \overline{S}_2\right)$. We show there is a consistent agreement that generates $\left(\overline{S}_1, \overline{S}_2\right)$, and so $\left(\overline{S}_1, \overline{S}_2\right) \in \mathcal{S}_{xy}$. Since $\left\{\left(S_1^n, S_2^n\right)\right\}_{n=1}^{\infty}$ converges, there exists N such that $\forall n > N$,

$$\max_{y'\in X}\left\{S^*\left(x,y'\right) : S^*\left(x,y'\right) < \overline{S}_1\right\} < S_1^n < \min_{y'\in X}\left\{S^*\left(x,y'\right) : S^*\left(x,y'\right) > \overline{S}_1\right\} \text{ and}$$
$$\max_{x'\in X}\left\{S^*\left(y,x'\right) : S^*\left(y,x'\right) < \overline{S}_2\right\} < S_2^n < \min_{x'\in X}\left\{S^*\left(y,x'\right) : S^*\left(y,x'\right) > \overline{S}_2\right\}.$$

Whenever $S_i^n > \overline{S}_i$ or $S_i^n < \overline{S}_i$, for $n > N$, $\widehat{d}_i^n$ is unique. We use this fact repeatedly in this proof.

We consider first the case where no $i \in \{1,2\}$ has $S_i^n = \overline{S}_i$ infinitely often. Then, there is a subsequence $\left\{\left(S_1^{n_m}, S_2^{n_m}\right)\right\}_{m=1}^{\infty}$ with either 1) $S_1^{n_m} > \overline{S}_1$ and $S_2^{n_m} > \overline{S}_2$, or 2) $S_1^{n_m} > \overline{S}_1$ and $S_2^{n_m} < \overline{S}_2$, or 3) $S_1^{n_m} < \overline{S}_1$ and $S_2^{n_m} > \overline{S}_2$, or finally 4) $S_1^{n_m} < \overline{S}_1$ and $S_2^{n_m} < \overline{S}_2$. In any such subsequence, for m big enough $\widehat{d}^{n_m} = \overline{d}$ is constant. So $S_1^{n_m}$ and $S_2^{n_m}$ are simply linear functions of $\widehat{\pi}_1^{n_m}$ and $\widehat{\pi}_2^{n_m}$. Since S^{n_m} converges, so does $\widehat{\pi}^{n_m} \to \overline{\pi}$. Moreover, since $\widehat{\pi}_1^{n_m} + \widehat{\pi}_2^{n_m} \leq f(x,y)\ \forall\ m$, then also $\overline{\pi}_1 + \overline{\pi}_2 \leq f(x,y)$. Thus, $\overline{c} = \left(\overline{d}, \overline{\pi}\right)$ generates $\left(\overline{S}_1, \overline{S}_2\right)$ and is consistent.

Next, we consider the case with $S_i^n = \overline{S}_i$ infinitely often for some $i \in \{1,2\}$. If $\left(S_1^n, S_2^n\right) = \left(\overline{S}_1, \overline{S}_2\right)$ for some n, then of course $\left(\overline{S}_1, \overline{S}_2\right) \in \mathcal{S}_{xy}$. Otherwise, without loss of generality, let $i = 1$. Then there is a subsequence $\left\{\left(S_1^{n_m}, S_2^{n_m}\right)\right\}_{m=1}^{\infty}$ with $S_1^{n_m} = \overline{S}_1$ and either always $S_2^{n_m} > \overline{S}_2$, or always $S_2^{n_m} < \overline{S}_2$. In any such subsequence for m big enough $\widehat{d}_2^{n_m} = \overline{d}_2$. Since $S_1^{n_m} = \overline{S}_1$, $\widehat{\pi}_1 = \overline{\pi}_1$ is also constant. Define $\overline{\pi}_2 = f(x,y) - \overline{\pi}_1 \geq \widehat{\pi}_2^n$. Let $\overline{d}_1$ be the most beneficial to 2 (so 1 does not leave if indifferent). Let $\widetilde{S} = \left(\overline{S}_1, \widetilde{S}_2\right)$ be induced by $\left(\overline{d}_1, \overline{d}_2\right)$ and $(\overline{\pi}_1, \overline{\pi}_2)$. $\overline{d}_1$ is no worse than what 2 gets in the subsequence, and $\overline{\pi}_2 \geq \widehat{\pi}_2^n$. Then $S_2^{n_m} \leq \widetilde{S}_2$. Thus, $\overline{S}_2 = \lim_{n\to\infty} S_2^n \leq \widetilde{S}_2$. If $\overline{S}_2 = \widetilde{S}_2$ we are done. Otherwise, decrease $\overline{\pi}_2$ to make it so. ■

A.2 Details on Multiplicity of Equilibria

Conditions 1 and 2 in our definition of equilibrium are not enough to weed out some fragile outcomes under on-the-match search.[17] Several divisions of output can satisfy these two conditions for a given decision function d^*, but not all of them are robust. Consider (d^*, π^*) satisfying conditions 1 and 2 and leading to equal surplus splitting in match (x,y). Take an alternative (d^*, π^{**}) with the *same* decision function and a small perturbation *only* in match (x,y)'s payoffs. Individual surpluses change only marginally, so agreements can still be consistent. Regarding condition 2, note that under the alternative (d^*, π^{**}) agent x expects $\pi^{**}(x,y)$ when matched to *any* type-y agent. If y offers x less than that, x breaks the match whenever she finds *another* type-y agent. Such an offer increases y's flow payoff marginally while the probability that x leaves increases discretely, making both partners worse off. To sum up, once $(\pi^{**}(x,y), \pi^{**}(y,x))$ is expected, any small deviation from it leads to a lower surplus for both partners. This example highlights that even keeping d^* and payoffs in all other matches fixed, several divisions of production in match (x,y) can satisfy conditions 1 and 2.

We include condition 3 to rule out fragile cases like (d^*, π^{**}), which would not survive a positive cost of transition. If breaking a match were costly, agent x would not leave for *another* type-y agent when receiving slightly less than the expected $\pi^{**}(x,y)$. So slight deviations from $(\pi^{**}(x,y), \pi^{**}(y,x))$ would increase the surplus of one agent while reducing the surplus of the other one *in the same amount* (as long as these slight deviations do not make agents leave for *other different types*). Thus, only symmetric surplus splitting would maximize the product of individual surpluses. Our third condition states that an agent can get a higher surplus than her partner only if she is indifferent between her current partner and a partner of a *different* type. This condition guarantees that equilibria are robust in the following sense.

Assume that agents have to pay a small cost $t > 0$ each time they quit their current partner to form a new match. Surplus from matches are then given by the

[17] In the Online Appendix we provide a simple example of how on-the-match search can lead to some uninteresting multiplicity of equilibrium.

following slightly modified version of (1):

$$S^*(x,y) = \left(r + \delta + \sum_{x' \in X} d^*(y,x,x')\, q^*(y,x') \right)^{-1} \Bigg[\pi^*(x,y)$$
$$+ \sum_{y' \in X} d^*(x,y,y')\, q^*(x,y') \left[S^*(x,y') - S^*(x,y) - t \right] - \sum_{y' \in X} q^*(x,y')\, S^*(x,y') \Bigg]$$

Take a pair (d^*, π^*) satisfying the first two conditions in our equilibrium definition. We show next that $S^*(x,y) > S^*(y,x) \Rightarrow \exists\, y' : S^*(x,y) = S^*(x,y') - t$ must be satisfied. Assume it is not. Then, there exists an alternative consistent agreement between x and y which leads to a higher product of individual surpluses. To build it, keep the decision function unchanged but pick $\tilde{\pi}(x,y) = \pi^*(x,y) - \varepsilon$ and $\tilde{\pi}(y,x) = \pi^*(y,x) + \varepsilon$. For small $\varepsilon > 0$, agent x does not change his behavior. Thus, the new pair $(d^*, \tilde{\pi})$ is consistent. Moreover, again for small $\varepsilon > 0$, the product of individual surpluses is larger. Then, the original pair (d^*, π^*) does not solve the bargaining problem.

A.3 Only Strict Heterophily with $d^*(h,h,\ell) = 1$

Lemma 3. $S^*(h,\ell) > S^*(h,h) \Rightarrow S^*(\ell,h) > S^*(\ell,\ell)$.

Proof. First, since $S^*(h,\ell) > S^*(h,h)$, the third condition in the equilibrium definition guarantees $S^*(\ell,h) \geq S^*(h,\ell)$. Next, consider the following alternative agreement for (h,h): they never leave each other and they split production. Let $\widehat{S}$ denote the surplus resulting from that agreement. Then,

$$S^*(h,\ell) \geq \widehat{S} = (r+\delta)^{-1} \left[h - q^*(h,\ell)\, S^*(h,\ell) - q^*(h,h) S^*(h,h) \right]$$

We show our result by contradiction. Assume $S^*(\ell,\ell) \geq S^*(\ell,h)$. Note that $q^*(\ell,h) \geq q^*(h,h)$ and $q^*(\ell,\ell) \geq q^*(h,\ell)$, since both agents prefer low types (at least weakly). Then,

$$S^*(\ell,\ell) = (r+\delta)^{-1} \left[\ell - q^*(\ell,\ell)\, S^*(\ell,\ell) - q^*(\ell,h) S^*(\ell,h) \right] < \widehat{S}$$

To sum up, $\widehat{S} > S^*(\ell,\ell) \geq S^*(\ell,h) \geq S^*(h,\ell) \geq \widehat{S}$. That is our contradiction. ■

A.4 Proof of Proposition 3

Surplus in equilibrium - as given by equation (1) - can be rewritten as

$$S^*(x,y) = \left(r + \delta + \sum_{x' \in X} d^*(y,x,x')\, q(y,x') \right)^{-1} \Bigg[\pi^*(x,y)$$

$$- \sum_{y' \in \overline{X}(y)} q\left(x,y'\right) S^{*}(x,y) - \sum_{y' \in \underline{X}(y)} q\left(x,y'\right) S^{*}\left(x,y'\right) \Bigg].$$

with $\overline{X}(y) = \{y' \in X : S^{*}(x,y') > S^{*}(x,y)\}$ and $\underline{X}(y) = \{y' \in X : S^{*}(x,y') \leq S^{*}(x,y)\}$. Note that $\sum_{y' \in \overline{X}(y)} q(x,y') S^{*}(x,y) + \sum_{y' \in \underline{X}(y)} q(x,y') S^{*}(x,y') \leq \sum_{y' \in X} q(x,y') S^{*}(x,y)$. Then,

$$(r+\delta+2\rho)^{-1} \pi^{*}(x,y) \leq S^{*}(x,y) \leq (r+\delta)^{-1} \pi^{*}(x,y). \tag{4}$$

Next, consider the following *simple* agreement for match (x,y): production is split evenly and both agents leave for any willing partner. Individual surpluses from such agreement are bounded below by $\widetilde{S} = (r+\delta+2\rho)^{-1} \frac{f(x,y)}{2}$. There exists a consistent agreement that gives more than $\widetilde{S}$ to both agents.[18] This - together with (4) - leads to the following bounds for the product of individual surpluses:

$$\left(\frac{r+\delta}{r+\delta+2\rho} \frac{f(x,y)}{2}\right)^{2} \leq (r+\delta)^{2} S^{*}(x,y) S^{*}(y,x) \leq \pi^{*}(x,y)\pi^{*}(y,x) \tag{5}$$

Payoffs approach an even split of production: $\lim_{\frac{\rho}{r+\delta} \to 0} \pi^{*}(x,y) = \frac{f(x,y)}{2}$, as equation (5) shows. Individual surplus are determined mainly by payoffs and the production function is strictly increasing, so $S^{*}(x,y+1) - S^{*}(x,y) > 0$ for small enough $\frac{\rho}{r+\delta}$. Thus, only hyperphily can be an equilibrium if $\frac{\rho}{r+\delta}$ is small.

Consider (d^{*}, π^{*}) under hyperphily. Condition 3 in our equilibrium definition is always satisfied for hyperphily since agents split surplus evenly. Condition 1 (consistency) is guaranteed for $\frac{\rho}{r+\delta}$ small, as shown above. Then, we only need to verify next that no consistent agreement c leads to a higher product of individual surpluses (condition 2).

Individual surpluses under hyperphily are given by

$$2\left(r+\delta+\sum_{x'>x} q^{*}\left(y,x'\right)+\sum_{y'>y} q^{*}\left(x,y'\right)\right) S^{*}(x,y) = f(x,y) - \sum_{y' \leq y} q^{*}\left(x,y'\right) S^{*}\left(x,y'\right) - \sum_{x' \leq x} q^{*}\left(y,x'\right) S^{*}\left(y,x'\right). \tag{6}$$

Let $\overline{F} = \max_{x,y} f(x,y)$. It is easy to find the following lower bound from (6):

$$2\left(r+\delta+2\rho\right) S^{*}(x,y) \geq f(x,y) - 2\overline{F}\frac{\rho}{r+\delta}$$

Any agreement leading to a higher product of individual must have $\widehat{S}_{1}^{c} \geq$

[18] Consider the simple agreement described. Calculate x's surplus. See who would x actually optimally leave for. Assume x behaves that way. Notice now y's surplus is weakly larger. Calculate y's best response now. At each step, neither x nor y can be worse off. So they leave each time for less people. Eventually, the process stops. That behavior is consistent.

$S^*(x, y+1)$ or $\widehat{S}_2^c \geq S^*(y, x+1)$ or both.[19] Assume without loss of generality that $\widehat{S}_1^c \geq S^*(x, y+1)$. Next, note that for any agreement $\widehat{S}_1^c + \widehat{S}_2^c \leq \frac{f(x,y)}{r+\delta}$. Again, pick $\frac{\rho}{r+\delta}$ small, so $S^*(x, y+1) \geq \frac{\frac{f(x,y)}{2}}{r+\delta}$. Then the product of surpluses must be bounded:

$$\widehat{S}_1^c \widehat{S}_2^c \leq \frac{\frac{f(x,y+1)}{2} - \overline{F}\frac{\rho}{r+\delta}}{r+\delta+2\rho}\left[\frac{f(x,y)}{r+\delta} - \frac{\frac{f(x,y+1)}{2} - \overline{F}\frac{\rho}{r+\delta}}{r+\delta+2\rho}\right]$$
$$< \left[\frac{\frac{f(x,y)}{2} - \overline{F}\frac{\rho}{r+\delta}}{r+\delta+2\rho}\right]^2 \leq S^*(x,y)S^*(y,x)$$

where again the last inequality holds for small $\frac{\rho}{r+\delta}$. ■

A.5 Hyperphily and Positive Assortative Matching

Proof. h's distribution of partners first order stochastically dominates ℓ's if and only if $e(\ell, \varnothing) > e(h, \varnothing)$ and $e(\ell, \varnothing) + e(\ell, \ell) > e(h, \varnothing) + e(h, \ell)$. Steady state conditions for $e(\ell, \varnothing)$ and $e(h, \varnothing)$ require, respectively, that:

$$\delta[\tfrac{1}{2} - e(\ell,\varnothing)] + \rho e(\ell,\ell)e(h,\varnothing) + \rho e(\ell,h)[e(h,\varnothing) + e(\ell,h)] = \rho e(\ell,\varnothing)[e(\ell,\varnothing) + e(h,\varnothing)] \tag{7}$$

$$\delta[\tfrac{1}{2} - e(h,\varnothing)] = \rho e(h,\varnothing)[e(\ell,\varnothing) + e(\ell,\ell) + e(h,\varnothing) + e(h,\ell)] \tag{8}$$

Then $e(\ell, \varnothing) > e(h, \varnothing)$. Otherwise, $e(\ell, \varnothing)$ and $e(h, \varnothing)$ cannot jointly be in steady state.[20] Next, consider steady state conditions for $e(\ell, h)$ and $e(h, h)$, respectively:

$$\rho[\tfrac{1}{2} - e(\ell,h)]e(h,\varnothing) = e(h,\ell)[\delta + \rho e(h,\varnothing) + \rho e(h,\ell)] \tag{9}$$

$$\rho[\tfrac{1}{2} - e(h,h)]^2 = \delta e(h,h) \tag{10}$$

So $e(h,h) > e(h,\ell)$. Otherwise, $e(\ell, \varnothing)$ and $e(h, \varnothing)$ cannot jointly be in steady state.[21] ■

A.5.1 The Two Type Case with Renegotiation

Renegotiation prevents inefficient separations. In the example presented in Section 5.2 individuals who meet unmatched agents switch partners as often as with

[19] To see this, note that if $\widehat{S}_1^c < S^*(x, y+1)$ and $\widehat{S}_2^c < S^*(y, x+1)$ then neither agent leaves the other less often. Then $\widehat{S}_1^c + \widehat{S}_2^c \leq 2S^*(x, y)$.

[20] Assume to the contrary that $e(\ell, \varnothing) \leq e(h, \varnothing)$. Then, the right hand side is lower in (7) than in (8), but the left hand side is lower in (8) than in (7).

[21] Assume to the contrary that $e(h, h) \leq e(h, \ell)$. Then, the right hand side is lower in (10) than in (9), but the left hand side is lower in (9) than in (10).

hyperphily with on-the-match-search. However, if the production function is not supermodular, as $2F > 2h$, a type-h agent matched to a ℓ-type one does not break the match when she finds a matched h-type agent. In contrast, in an equilibrium featuring hyperphily with on-the-match-search, h leaves ℓ if she meets another h matched to ℓ. Therefore the steady state distribution of matches in a model with and without renegotiation may differ.[22] However, it is still straightforward to show that h's distribution of partners first order stochastically dominates ℓ's.

With renegotiation, steady state conditions for $e(\ell,\varnothing), e(h,\varnothing), e(\ell,h)$, and $e(h,h)$ now require, respectively, that:

$$\delta[\tfrac{1}{2} - e(\ell,\varnothing)] + \rho e(\ell,\ell)e(h,\varnothing) + \rho e(\ell,h)e(h,\varnothing) = \rho e(\ell,\varnothing)[e(\ell,\varnothing) + e(h,\varnothing)]$$
$$\delta[\tfrac{1}{2} - e(h,\varnothing)] = \rho e(h,\varnothing)[e(\ell,\varnothing) + e(\ell,\ell) + e(h,\varnothing) + e(h,\ell)]$$
$$\rho[\tfrac{1}{2} - e(\ell,h)]e(h,\varnothing) = e(h,\ell)[\delta + \rho e(h,\varnothing)]$$
$$\rho[\tfrac{1}{2} - e(h,h)]e(h,\varnothing) + \rho e(h,\varnothing)e(h,\ell) = \delta e(h,h)$$

A proof analogous to that without renegotiation shows that h's distribution of partners first order stochastically dominates ℓ's.

A.6 Conditions for Hyperphily

Under hyperphily $d(\ell,\ell,h) = d(h,\ell,h) = 1$ and $d(\ell,h,\ell) = d(h,h,\ell) = 0$. Then, the steady state conditions become:

$$\begin{aligned}
e(\ell,\ell)\left[\delta + q(\ell,h)\right] + e(\ell,h)\left[\delta + q(h,h)\right] &= e(\ell,\varnothing)\left[q(\ell,\ell) + q(\ell,h)\right] \\
e(\ell,\varnothing)q(\ell,\ell) &= e(\ell,\ell)\left[\delta + 2q(\ell,h)\right] \\
\left[e(\ell,\varnothing) + e(\ell,\ell)\right] q(\ell,h) &= e(\ell,h)\left[\delta + q(h,h)\right] \\
\delta\left[e(h,\ell) + e(h,h)\right] &= e(h,\varnothing)\left[q(h,\ell) + q(h,h)\right] \\
\left[e(h,\varnothing) + e(h,\ell)\right] q(h,h) &= \delta e(h,h)
\end{aligned}$$

The successful meeting rates become, $q(\ell,\ell) = \rho e(\ell,\varnothing), q(\ell,h) = \rho e(h,\varnothing), q(h,\ell) = \rho[e(\ell,\varnothing) + e(\ell,\ell)]$ and $q(h,h) = \rho[e(h,\varnothing) + e(h,\ell)]$. Substituting these into the steady state conditions, dividing by ρ, and setting $\kappa = \frac{\rho}{\delta}$, we get

$$e(\ell,\ell)\left[\kappa^{-1} + e(h,\varnothing)\right] + e(\ell,h)\left[\kappa^{-1} + \tfrac{1}{2} - e(h,h)\right] = e(\ell,\varnothing)\left[e(\ell,\varnothing) + e(h,\varnothing)\right] \tag{11}$$

$$e(\ell,\varnothing)^2 = e(\ell,\ell)\left[\kappa^{-1} + 2e(h,\varnothing)\right] \tag{12}$$

$$\left[e(\ell,\varnothing) + e(\ell,\ell)\right] e(h,\varnothing) = e(\ell,h)\left[\kappa^{-1} + e(h,\varnothing) + e(h,\ell)\right] \tag{13}$$

[22] If only one side of the market searches on the match, the distribution of matches in a model with or without renegotiation is the same (see Bartolucci [2013]). This is because a matched agent can never meet another matched agent.

$$\kappa^{-1}\left[e(h,\ell)+e(h,h)\right]=e(h,\varnothing)\left[e(\ell,\varnothing)+e(\ell,\ell)+e(h,\varnothing)+e(h,\ell)\right] \tag{14}$$

$$\left[e(h,\varnothing)+e(h,\ell)\right]^2=\kappa^{-1}e(h,h) \tag{15}$$

Solving equation (14) for $e(h,\varnothing)$, gives a quadratic equation in the unknown $e(h,\varnothing)$, $\kappa^{-1}\left[\frac{1}{2}-e(h,\varnothing)\right]=e(h,\varnothing)\left[\frac{1}{2}+e(h,\varnothing)\right]$, the positive solution of which is

$$e(h,\varnothing)=\frac{1}{2}\left(\sqrt{\kappa^{-2}+3\kappa^{-1}+\frac{1}{4}}-\kappa^{-1}-\frac{1}{2}\right)$$

A similar procedure on equation (15), gives

$$e(h,h)=\frac{1}{2}\left(1+\kappa^{-1}-\sqrt{\kappa^{-2}+2\kappa^{-1}}\right)$$

Using these two results together with the normalization condition gives

$$e(h,\ell)=e(\ell,h)=\frac{1}{2}\left(\frac{1}{2}+\sqrt{\kappa^{-2}+2\kappa^{-1}}-\sqrt{\kappa^{-2}+3\kappa^{-1}+\frac{1}{4}}\right)$$

Solving equation (11) for $e(\ell,\varnothing)$ yields the following quadratic equation (the $e(h,\cdot)$ are all known by now):

$$e(\ell,\varnothing)^2+\left[\kappa^{-1}+2e(h,\varnothing)\right]e(\ell,\varnothing)-\frac{1}{2}\kappa^{-1}-\frac{1}{2}e(h,\varnothing)-e(h,\ell)^2=0$$

Its positive solution, after substituting in the values of $e(h,\varnothing)$ and $e(h,\ell)$, is

$$\begin{aligned}e(\ell,\varnothing)=&\frac{1}{4}-\frac{1}{2}\sqrt{\kappa^{-2}+3\kappa^{-1}+\frac{1}{4}}\\&+\frac{1}{2}\sqrt{3\kappa^{-2}+9\kappa^{-1}+\frac{1}{2}-\sqrt{\kappa^{-2}+3\kappa^{-1}+\frac{1}{4}}\left(2\sqrt{\kappa^{-2}+2\kappa^{-1}}+1\right)+\sqrt{\kappa^{-2}+2\kappa^{-1}}}\end{aligned}$$

Finally, since $e(\ell,\ell)=\frac{1}{2}-e(\ell,\varnothing)-e(h,\ell)$ we get

$$\begin{aligned}e(\ell,\ell)=&\sqrt{\kappa^{-2}+3\kappa^{-1}+\frac{1}{4}}-\frac{1}{2}\sqrt{\kappa^{-2}+2\kappa^{-1}}\\&-\frac{1}{2}\sqrt{3\kappa^{-2}+9\kappa^{-1}+\frac{1}{2}-\sqrt{\kappa^{-2}+3\kappa^{-1}+\frac{1}{4}}\left(2\sqrt{\kappa^{-2}+2\kappa^{-1}}+1\right)+\sqrt{\kappa^{-2}+2\kappa^{-1}}}\end{aligned}$$

Under hyperphily, surpluses are:

$$S^*(h,\ell)=\left[r+\delta+q^*(h,\ell)+q^*(h,h)\right]^{-1}\left[F-\pi^*(\ell,h)\right]$$

$$S^*(h,h) = [r+\delta+q^*(h,h)]^{-1}[h - q^*(h,\ell)S^*(h,\ell)]$$
$$S^*(\ell,h) = [r+\delta+q^*(\ell,h)+q^*(h,h)]^{-1}[\pi^*(\ell,h) - q^*(\ell,\ell)S^*(\ell,\ell)]$$
$$S^*(\ell,\ell) = [r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)]^{-1}\ell$$

Surplus equalization $S^*(h,\ell) = S^*(\ell,h)$ requires:

$$\pi^*(\ell,h) = \frac{[r+\delta+q^*(\ell,h)+q^*(h,h)]\,F + \frac{r+\delta+q^*(h,\ell)+q^*(h,h)}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}q^*(\ell,\ell)\ell}{2\,(r+\delta+q^*(h,h))+q^*(\ell,h)+q^*(h,\ell)}$$

As discussed in Section 3.1, a pair (d^*,π^*) is consistent in an equilibrium with hyperphily if $G^1_{HYP} \equiv S^*(h,h) - S^*(h,\ell) > 0$ and $G^2_{HYP} \equiv S^*(\ell,h) - S^*(\ell,\ell) > 0$.

Bargaining in match (ℓ,h)

Condition 1 in the main text lists the three kinds of agreement which may prevent (d^*,π^*) from solving the bargaining problem in the match (ℓ,h). We check next when these agreements are not feasible. First, in agreement c_1, neither ℓ nor h leave each other, and h is made indifferent. ℓ obtains $\widehat{S}^{c_1}_\ell$. We need then $G^3_{HYP} \equiv S^*(\ell,h) - \widehat{S}^{c_1}_\ell > 0$. In agreement c_2, $\widehat{S}^{c_2}_h = S^*(h,h)$ (thus h never leaves) and ℓ only leaves when she finds an h, leading to surplus $\widehat{S}^{c_2}_\ell$. We need then $G^4_{HYP} \equiv S^*(\ell,h)S^*(h,\ell) - \widehat{S}^{c_2}_\ell S^*(h,h) \geq 0$. Finally, agreement c_3 also has $\widehat{S}^{c_3}_h = S^*(h,h)$, but now ℓ always leaves. We need $G^5_{HYP} \equiv S^*(\ell,h)S^*(h,\ell) - \widehat{S}^{c_3}_\ell S^*(h,h) \geq 0$.

Bargaining in match (ℓ,ℓ)

Condition 2 lists the three kinds of agreements which may prevent (d^*,π^*) from solving the bargaining problem in match (ℓ,ℓ). We check next when these agreements are not feasible. First, let $\widehat{S}^{c_4}_1$ be the surplus obtained by either agent in match (ℓ,ℓ) when they do not leave each other. If c_4 were consistent, it would lead to a higher product of surpluses, as both agents would receive a higher surplus. Therefore $G^6_{HYP} \equiv S^*(\ell,h) - \widehat{S}^{c_4}_1 > 0$ must hold for hyperphily to be an equilibrium. Next, in agreement c_5 one agent ℓ obtains $S^*(\ell,h)$ and does not leave, whereas the other one leaves only when meeting agent h. We need then $G^7_{HYP} \equiv S^*(\ell,\ell) - \widehat{S}^{c_5}_1 > 0$. Finally, in agreement c_6, $\widehat{S}^{c_6}_1$ is the surplus obtained by an ℓ agent in match (ℓ,ℓ) when she always leaves and her partner is indifferent between this match and one with h. For hyperphily to solve the bargaining problem, it must be the case that $G^8_{HYP} \equiv S^*(\ell,\ell)^2 - \widehat{S}^{c_6}_1\widehat{S}^{c_6}_2 \geq 0$.

Bargaining in match (h,h)

In match (h,h) there is no endogenous destruction and agents equalize surplus, therefore they are maximizing the product of surpluses.

Details on equilibrium conditions

We characterize each condition as a function of primitives. Let us start with match (ℓ, h). In alternative agreement c_1 for match (ℓ, h), we have $\widehat{d}_\ell = \widehat{d}_h = 0$ and $\widehat{\pi}_h = h$. Note that for any lower $\widehat{\pi}_h$, h leaves. Next, any higher $\widehat{\pi}_h$ leads to a lower product of surpluses. It suffices then to focus on this agreement. We need to show that $G^3_{HYP} \equiv S^*(\ell, h) - \widehat{S}^{c_1}_\ell > 0$.

$$\widehat{S}^{c_1}_\ell = (r+\delta)^{-1}\left[F - h - q^*(\ell,\ell)S^*(\ell,\ell) - q^*(\ell,h)S^*(\ell,h)\right] \quad \text{and}$$

$$S^*(\ell,h) = \frac{F - \frac{q^*(\ell,\ell)\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}}{2\left(r+\delta+q^*(h,h)\right)+q^*(\ell,h)+q^*(h,\ell)}$$

Therefore, we need

$$F - h - \frac{q^*(\ell,\ell)\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)} < \frac{[r+\delta+q^*(\ell,h)]\left[F - \frac{q^*(\ell,\ell)\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\right]}{2[r+\delta+q^*(h,h)]+q^*(\ell,h)+q^*(h,\ell)}$$

Thus

$$F < h\left(1 + \frac{r+\delta+q^*(\ell,h)}{r+\delta+2q^*(h,h)+q^*(h,\ell)}\right) + \frac{q^*(\ell,\ell)\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)} \quad \text{(HYP 1)}$$

Next, in agreement c_2, $\widehat{d}_h = 0$, $\widehat{d}_\ell(\ell) = 0$, $\widehat{d}_\ell(h) = 1$ and $\widehat{\pi}_\ell$ is such that $\widehat{S}^{c_2}_h = S^*(h,h)$. Note that for any lower $\widehat{\pi}_h = F - \widehat{\pi}_\ell$, h leaves. Next, any higher $\widehat{\pi}_h$ leads to a lower product of surpluses. It suffices then to focus on this agreement. We need to show that $G^4_{HYP} \equiv S^*(\ell,h)S^*(h,\ell) - \widehat{S}^{c_2}_\ell \widehat{S}^{c_2}_h \geq 0$. Surpluses are:

$$\widehat{S}^{c_2}_h = (r+\delta+q^*(\ell,h))^{-1}\left(F - \widehat{\pi}_\ell - q^*(h,h)S^*(h,h) - q^*(h,\ell)S^*(h,\ell)\right) = S^*(h,h)$$
$$\widehat{S}^{c_2}_\ell = (r+\delta+q^*(\ell,h))^{-1}\left(\widehat{\pi}_\ell - q^*(\ell,\ell)S^*(\ell,\ell)\right)$$

From this we can recover $\widehat{\pi}_\ell$:

$$\widehat{\pi}_\ell = F - [r+\delta+q^*(h,h)]^{-1}\left[\left(r+\delta+q^*(\ell,h)+q^*(h,h)\right)h - \frac{q^*(h,\ell)q^*(\ell,h)(F-\pi^*(\ell,h))}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right]$$

From now on, we work with $\pi^*(\ell,h) = A_1 + B_1 F$ and $\widehat{\pi}_\ell = A_2 + B_2 F$, with:

$$A_1 = \frac{r+\delta+q^*(h,\ell)+q^*(h,h)}{[r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)]\left[2\left(r+\delta+q^*(h,h)\right)+q^*(\ell,h)+q^*(h,\ell)\right]}q^*(\ell,\ell)\ell$$
$$B_1 = \frac{r+\delta+q^*(\ell,h)+q^*(h,h)}{2\left(r+\delta+q^*(h,h)\right)+q^*(\ell,h)+q^*(h,\ell)}$$
$$A_2 = -\frac{r+\delta+q^*(\ell,h)+q^*(h,h)}{r+\delta+q^*(h,h)}h - \frac{q^*(h,\ell)q^*(\ell,h)A_1}{\left(r+\delta+q^*(h,\ell)+q^*(h,h)\right)\left(r+\delta+q^*(h,h)\right)}$$

$$B_2 = 1 + \frac{q^*(h,\ell)q^*(\ell,h)(1-B_1)}{(r+\delta+q^*(h,h))\,(r+\delta+q^*(h,\ell)+q^*(h,h))}$$

We first check that $\widehat{S}_\ell^{c_2} \geq S^*(\ell,\ell)$. This occurs whenever

$$F \geq B_2^{-1}\left(\frac{r+\delta+q^*(\ell,h)+q^*(\ell,\ell)}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\ell - A_2\right)$$

If agreement c_2 is consistent we still have to check that F is large enough to make the product of surpluses larger, that is, $S^*(h,\ell)S^*(\ell,h) \geq \widehat{S}_h^{c_2}\widehat{S}_\ell^{c_2}$:

$$\begin{aligned}
\left[(1-B_1)F - A_1\right]^2 \geq C_1 &\left(B_2F + A_2 - \frac{q^*(\ell,\ell)\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\right)\\
&\times\left(h - q^*(h,\ell)\frac{(1-B_1)F - A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right) \quad \text{with}\\
C_1 = &\frac{(r+\delta+q^*(h,h)+q^*(h,\ell))^2}{(r+\delta+q^*(\ell,h))(r+\delta+q^*(h,h))}.
\end{aligned}$$

The previous expression holds with equality for F given by:

$$\begin{aligned}
&\left[(1-B_1)^2 + C_1B_2\frac{q^*(h,\ell)(1-B_1)}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right]F^2\\
&+\left[-2(1-B_1)A_1 - C_1B_2\left(h + \frac{q^*(h,\ell)A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right)\right]F\\
&+\frac{C_1q^*(h,\ell)(1-B_1)}{r+\delta+q^*(h,\ell)+q^*(h,h)}\left(A_2 - \frac{q^*(\ell,\ell)\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\right)F\\
&+A_1^2 - C_1\left(h + \frac{q^*(h,\ell)A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right)\left(A_2 - \frac{q^*(\ell,\ell)\ell}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\right) = 0
\end{aligned}$$

Since $(1-B_1)$, C_1 and B_2 are positive, G^4_{HYP} is a convex function of F. In order to have an equilibrium with hyperphily, F has to be smaller than the lower root or larger than the higher one. Only the first of these two conditions is relevant. To see this, note that there exists an $\widehat{F}$ such that $S^*(\ell,h) = S^*(h,h)$. For $F = \widehat{F}$, $\widehat{S}_\ell^{c_2}\widehat{S}_h^{c_2} > S^*(\ell,h)S^*(h,\ell)$ holds.[23] Therefore, F larger than the large root of $G^4_{HYP} = 0$ requires that $F > \widehat{F}$. However, consistency condition G^1_{HYP} states than an equilibrium with hyperphily requires $F < \widehat{F}$. Therefore if F^4_{HYP} is the small root of $G^4_{HYP} = 0$, an

[23] If $F = \widehat{F}$, this is equivalent to $\widehat{S}_\ell^{c_2} > S^*(\ell,h)$ because $\widehat{S}_h^{c_2} = S^*(h,h) = S^*(h,\ell)$. Add $\widehat{S}_h^{c_2} = S^*(h,\ell)$ on both sides of the inequality and rearrange terms to get:

$$\begin{aligned}
&\frac{\widehat{F} - q^*(h,h)S^*(h,h) - q^*(h,\ell)S^*(h,\ell) - q^*(\ell,\ell)S^*(\ell,\ell)}{r+\delta+q^*(\ell,h)}\\
&> \frac{\widehat{F} + [q^*(\ell,h) - q^*(h,\ell) - q^*(h,h)]S^*(h,\ell) - q^*(\ell,\ell)S^*(\ell,\ell) - q^*(h,h)S^*(\ell,h)}{r+\delta+q^*(\ell,h)}
\end{aligned}$$

Comparing numerators gives $q^*(h,h) > q^*(\ell,h)$, which indeed holds.

equilibrium with hyperphily requires:

$$F \leq \max\left(F^4_{HYP}, B_2^{-1}\left[\frac{r+\delta+q^*(\ell,h)+q^*(\ell,\ell)}{r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)}\ell - A_2\right]\right) \tag{HYP 2}$$

We move next to alternative agreement c_3. Surpluses are:

$$\widehat{S}^{c_3}_h = [r+\delta+q^*(\ell,h)+q^*(\ell,\ell)]^{-1}\left[F-\widehat{\pi}_\ell - q^*(h,h)S^*(h,h)-q^*(h,\ell)S^*(h,\ell)\right] = S^*(h,h)$$
$$\widehat{S}^{c_3}_\ell = [r+\delta+q^*(\ell,h)+q^*(\ell,\ell)]^{-1}\widehat{\pi}_\ell$$

Then $\widehat{\pi}_\ell = A_3 + B_3 F$, with

$$A_3 = -\frac{r+\delta+q^*(\ell,h)+q^*(\ell,\ell)+q^*(h,h)}{r+\delta+q^*(h,h)}h - \frac{q^*(h,\ell)(q^*(\ell,h)+q^*(\ell,\ell))A_1}{[r+\delta+q^*(h,h)][r+\delta+q^*(h,h)+q^*(h,\ell)]}$$
$$B_3 = 1 + \frac{q^*(h,\ell)(q^*(\ell,h)+q^*(\ell,\ell))B_1}{[r+\delta+q^*(h,h)]\,[r+\delta+q^*(h,h)+q^*(h,\ell)]}$$

Condition $G^5_{HYP} \equiv S^*(h,\ell)S^*(\ell,h) - \widehat{S}^{c_3}_h\widehat{S}^{c_3}_\ell \geq 0$ holds if:

$$[(1-B_1)F-A_1]^2 \geq C_2\,(B_3F+A_3)\left(h-q^*(h,\ell)\frac{(1-B_1)F-A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right) \quad \text{with}$$
$$C_2 = \frac{(r+\delta+q^*(h,h)+q^*(h,\ell))^2}{(r+\delta+q^*(\ell,h)+q^*(\ell,\ell))(r+\delta+q^*(h,h))}$$

Therefore:

$$\begin{aligned} G^5_{HYP} = &\left[(1-B_1)^2 + C_2B_3\frac{q^*(h,\ell)(1-B_1)}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right]F^2 \\ &+\left[-2(1-B_1)A_1 - C_2B_3\left(h+\frac{q^*(h,\ell)A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right)+\frac{q^*(h,\ell)(1-B_1)}{r+\delta+q^*(h,\ell)+q^*(h,h)}C_2A_3\right]F \\ &+A_1^2 - \left(h+\frac{q^*(h,\ell)A_1}{r+\delta+q^*(h,\ell)+q^*(h,h)}\right)C_2A_3 \geq 0 \end{aligned}$$

Since $(1-B_1)$, C_2, and B_3 are positive, G^6_{HYP} is a convex function of F. There are two values $F^5_{HYP} < F^{5'}_{HYP}$ of F that equalize the product of the surplus. In order to have an equilibrium with hyperphily F has to be smaller than F^5_{HYP} or larger then $F^{5'}_{HYP}$:

$$F \notin \left(F^5_{HYP}, F^{5'}_{HYP}\right) \tag{HYP 3}$$

We move next to match (ℓ,ℓ). Consider alternative agreement c_4. We need to show that $G^6_{HYP} \equiv S^*(\ell,h) - \widehat{S}^{c_4}_1 > 0$, with

$$\widehat{S}^{c_4}_1 = (r+\delta)^{-1}\left[\ell - q^*(\ell,h)S^*(\ell,h) - q^*(\ell,\ell)S^*(\ell,\ell)\right]$$

This occurs whenever

$$F > \frac{[2(r+\delta+q^*(h,h))+q^*(\ell,h)+q^*(h,\ell)][r+\delta+2q^*(\ell,h)]+[r+\delta+q^*(\ell,h)]q^*(\ell,\ell)}{r+\delta+q^*(\ell,h)][r+\delta+q^*(\ell,\ell)+2q^*(\ell,h)]}\ell \tag{HYP 4}$$

Next, consider agreement c_5. Agent ℓ indexed by 2 does not leave so $\widehat{S}_2^{c_5} = S^*(\ell,h)$, with

$$\widehat{S}_2^{c_5} = (r+\delta+q^*(\ell,h)^{-1}[\widehat{\pi}_2 - q^*(\ell,\ell)S^*(\ell,\ell) - q^*(\ell,h)S^*(\ell,h)]$$

This requires

$$\begin{aligned}\widehat{\pi}_2 = & \frac{r+\delta+2q^*(\ell,h)}{2\left(r+\delta+q^*(h,h)\right)+q^*(\ell,h)+q^*(h,\ell)}F \\ & + \frac{r+\delta+2q^*(h,h)-q^*(\ell,h)+q^*(h,\ell)}{[r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)][2(r+\delta+q^*(h,h))+q^*(\ell,h)+q^*(h,\ell)]}q^*(\ell,\ell)\ell\end{aligned}$$

We need to verify now that $G^7_{HYP} \equiv S^*(\ell,\ell) - \widehat{S}_1^{c_5} > 0$ with

$$\widehat{S}_1^{c_5} = \frac{2\ell - \widehat{\pi}_2 - q^*(\ell,\ell)S^*(\ell,\ell)}{r+\delta+q^*(\ell,h)} < S^*(\ell,\ell)$$

If condition G^6_{HYP} holds, it must be the case that $\widehat{S}_1^{c_5} < S^*(\ell,h)$. We look for the maximum F that makes the agreement $\left(\widehat{S}_1^{c_5}, \widehat{S}_2^{c_5}\right)$ consistent:

$$\begin{aligned}F > & \frac{[r+\delta+3q^*(\ell,h)+q^*(\ell,\ell)][2(r+\delta+q^*(h,h))+q^*(\ell,h)+q^*(h,\ell)]}{[r+\delta+2q^*(\ell,h)][r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)]} \\ & - \frac{[r+\delta+2q^*(h,h)-q^*(\ell,h)+q^*(h,\ell)]}{[r+\delta+2q^*(\ell,h)][r+\delta+2q^*(\ell,h)+q^*(\ell,\ell)]}\ell\end{aligned} \tag{HYP 5}$$

Next, consider agreement c_6. We need to verify that $G^8_{HYP} \equiv S^*(\ell,\ell)^2 - \widehat{S}_1^{c_6}\widehat{S}_2^{c_6} \geq 0$ with

$$\widehat{S}_1^{c_6} = \frac{2\ell - \widehat{\pi}_2}{r+\delta+q^*(\ell,h)+q^*(\ell,\ell)} \quad \text{and}$$

$$\widehat{S}_2^{c_6} = \frac{\widehat{\pi}_2 - q^*(\ell,h)S^*(\ell,h) - q^*(\ell,\ell)S^*(\ell,\ell)}{r+\delta+q^*(\ell,\ell)+q^*(\ell,h)}$$

Note that

$$\widehat{S}_1^{c_6} + \widehat{S}_2^{c_6} = \frac{2\ell - q^*(\ell,h)S^*(\ell,h) - q^*(\ell,\ell)S^*(\ell,\ell)}{r+\delta+q^*(\ell,\ell)+q^*(\ell,h)} \quad \text{and}$$

$$2S^*(\ell,\ell) = \frac{2\ell}{r+\delta+q^*(\ell,\ell)+2q^*(\ell,h)} = \frac{2\ell - 2q^*(\ell,h)S^*(\ell,\ell)}{r+\delta+q^*(\ell,\ell)+q^*(\ell,h)}$$

Since $S^*(\ell,\ell) < S^*(\ell,h)$ and $q^*(\ell,\ell) = \rho e(\ell,\varnothing) > \rho e(h,\varnothing) = q^*(\ell,h)$, then $2S^*(\ell,\ell) > \widehat{S}_1^{c_6} + \widehat{S}_2^{c_6}$. Both ℓ agents equalize surplus in $S^*(\ell,\ell)$, and no agreement in the same segment of the frontier or in an interior segment of the frontier can generate a larger product of surpluses. Therefore condition G_{HYP}^8 always holds.

Finally, we check consistency of the equilibrium with hyperphily. For condition G_{HYP}^1, note that $S^*(h,h) > S^*(h,\ell)$ if and only if $h > \pi^*(h,\ell)$. The following expression holds: $S^*(\ell,h) - \widehat{S}_\ell^{c_1} = (r+\delta)^{-1}\left[\pi^*(\ell,h) - (F-h) - q^*(h,h)S^*(\ell,h)\right]$. Condition G_{HYP}^3 implies the previous expression is positive. Thus $\pi^*(\ell,h) - F + h > 0 \Rightarrow F - \pi^*(h,\ell) - F + h > 0 \Rightarrow h > \pi^*(h,\ell)$, so G_{HYP}^1 holds.

G_{HYP}^2 holds whenever G_{HYP}^6 holds. To see this, note that

$$\widehat{S}_1^{c_4} = S^*(\ell,\ell) + \frac{q^*(\ell,h)\left[2S^*(\ell,\ell) - S^*(\ell,h)\right]}{r+\delta}.$$

Then, whenever $S^*(\ell,\ell) - S^*(\ell,h) \geq 0$, also $\widehat{S}_1^{c_4} - S^*(\ell,h) \geq 0$.

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