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Enlarging minimal-supergravity parameter space by decreasing pre-nucleosynthesis Hubble rate in scalar-tensor cosmologies

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Abstract: We determine under what conditions Scalar Tensor cosmologies predict an expansion rate which is reduced as compared to the standard General Relativity case. We show that ST theories with a single matter sector typically predict an enchanced Hubble rate in the past, as a consequence of the requirement of an attractive fixed point towards General Relativity at late times. Instead, when additional matter sectors with different conformal factors are added, the late time convergence to General Relativity is mantained and at the same time a reduced expansion rate in the past can be driven. For suitable choices of the parameters which govern the scalar field evolution, a sizeable reduction of the Hubble rate prior to Big Bang Nucleosynthesis can be obtained. We then discuss the impact of these cosmological models on the relic abundance of dark matter in minimal Supergravity models: we show that the cosmologically allowed regions in parameter space are significantly enlarged in this class of ST theories with reduced Hubble rate. The ensuing effect on the potential reach of LHC are biefly discussed.

Keywords: Cosmology of Theories beyond the SM, Classical Theories of Gravity, Supergravity Models, Supersymmetry Phenomenology.

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Contents

1. Changing the expansion rate in the past

In a standard flat FRW universe described by GR, the expansion rate of the universe, $H_{\rm GR} \equiv \dot{a}/a$, is set by the total energy density, $\tilde{\rho}_{\rm tot}$, according to the Friedmann law,

$$H^2_{\rm GR} = \frac{1}{3M_p^2}\,\tilde{\rho}_{\rm tot}\,, \tag{1.1}$$

where M_p is the Planck mass, related to the Newton constant by $M_p = (8\pi G)^{-1/2}$. If the total energy density is dominated by relativistic degrees of freedom, the expansion rate is related to the temperature through the relation

$$H_{\rm GR} \simeq 1.66\, g_*^{1/2}\,\frac{T^2}{M_p}\,, \tag{1.2}$$

with g_* the effective number of relativistic degrees of freedom (see for instance [1]).

In order to modify the above H–T relation, one can do one (or more) of the following:

1) change the number of relativistic d.o.f.'s, g_*;

2) consider a $\tilde{\rho}_{\rm tot}$ not dominated by relativistic d.o.f.'s;

3) consider a modification of GR in which an effective Planck mass, different from M_p appears in (1.2).

One example of a scenario of the first type is obtained by adding N extra light neutrino families to the standard model, which increases g_* by $7/4\,N$.

The second situation is realized e.g. in the so called "kination" scenario [2], where the energy density at a certain epoch is dominated by the kinetic energy of a scalar field. Since the kinetic energy redshifts as $\rho_{\rm kin} \sim a^{-6}$, it will eventually become subdominant with respect to radiation ($\rho_{\rm rad} \sim a^{-4}$). As long as $\rho_{\rm kin}$ dominates, the expansion rate is bigger than in the "standard" scenario where there is no scalar field and the same amount of radiation.

In this paper we will consider scenarios of the third type, where the expansion rate is modified by changing the effective gravitational coupling. This can be realized in a fully covariant way in ST theories [3]. We will consider the class of ST theories which can be defined by the following action [4],

$$S = S_g + \sum_i S_i, \tag{1.3}$$

where S_g is the gravitational part, given by the sum of the Einstein-Hilbert and the scalar field actions,

$$S_g = \frac{M_*^2}{2} \int d^4x \sqrt{-g} \left[R + g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{2}{M_*^2} V(\varphi) \right], \tag{1.4}$$

where $V(\varphi)$ can be either a true potential or a (Einstein frame) cosmological constant, $V(\varphi) = V_0$. The S_i's are the actions for separate "matter" sectors

$$S_i = S_i[\Psi_i, A_i^2(\varphi) g_{\mu\nu}] \ , \tag{1.5}$$

with Ψ_i indicating a generic field of the i-th matter sector, coupled to the metric $A_i^2(\varphi) g_{\mu\nu}$. The actions S_i are constructed starting from the Minkowski actions of Quantum Field Theory, for instance the SM or the MSSM ones, by substituting the flat metric $\eta_{\mu\nu}$ everywhere with $A_i^2(\varphi) g_{\mu\nu}$.

The emergence of such a structure, with different conformal factors A_i^2 for the various sectors can be motivated in extra-dimensional models, assuming that the two sectors live in different portions of the extra-dimensional space. A similar structure, leading to more dark matter species, each with a different conformal factor, was considered in [5].

We consider a flat FRW space-time

$$ds^2 = dt^2 - a^2(t)\, dl^2 \,,$$

where the matter energy-momentum tensors, $T^i_{\mu\nu} \equiv 2(-g)^{-1/2} \delta S_i / \delta g^{\mu\nu}$ admit the perfect-fluid representation

$$T^i_{\mu\nu} = (\rho_i + p_i)\, u_\mu u_\nu \ - p_i\, g_{\mu\nu} \ , \tag{1.6}$$

with $g_{\mu\nu}\, u^\mu u^\nu = 1$.

The cosmological equations then take the form

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_*^2} \left[\sum_i (\rho_i + 3\, p_i) + 2M_*^2 \dot{\varphi}^2 - 2V \right], \tag{1.7}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_*^2} \left[\sum_i \rho_i + \frac{M_*^2}{2} \dot{\varphi}^2 + V \right], \tag{1.8}$$

$$\ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} = -\frac{1}{M_*^2} \left[\sum_i \alpha_i (\rho_i - 3p_i) + \frac{\partial V}{\partial \varphi} \right], \tag{1.9}$$

where the coupling functions α_i are given by

$$\alpha_i \equiv \frac{d \log A_i}{d\varphi} \,. \tag{1.10}$$

The Bianchi identity holds for each matter sector separately, and reads,

$$d(\rho_i\, a^3) + p_i\, da^3 = (\rho_i - 3\, p_i)\; a^3 d\log A_i(\varphi), \tag{1.11}$$

implying that the energy densities scale as

$$\rho_i \sim A_i(\varphi)^{1-3w_i} a^{-3(1+w_i)} \,, \tag{1.12}$$

with $w_i \equiv p_i/\rho_i$ the equation of state associated to the i-th energy density (assuming w_i is constant).

1.1 GR as a fixed point

To start, consider the case of a single matter sector, S_M. In order to compare the ST case with the GR one of eqs. (1.1), (1.2), it is convenient to Weyl-transform to the so-called Jordan Frame (JF), where the energy-momentum tensor is covariantly conserved. The transformation amounts to a rescaling of the metric according to

$$\tilde{g}_{\mu\nu} = A_M^2(\varphi) g_{\mu\nu} \,, \tag{1.13}$$

keeping the comoving spatial coordinates and the conformal time $d\eta = dt/a$ fixed [6]. The JF matter energy-momentum tensor, $\tilde{T}^M_{\mu\nu} \equiv 2(-\tilde{g})^{-1/2}\delta S_M/\delta\tilde{g}^{\mu\nu}$, is related to that in eq. (1.6) by $\tilde{T}^M_{\mu\nu} = A_M^{-2} T^M_{\mu\nu}$, so that energy density and pressure transform as

$$\tilde{\rho}_M = A_M^{-4}\rho_M \,, \qquad\qquad \tilde{p}_M = A_M^{-4} p_M \,, \tag{1.14}$$

while the cosmic time transforms as $d\tilde{t} = A_M dt$. One can easily verify that the above defined quantities satisfy the usual Bianchi identity, that is eq. (1.11) with vanishing r.h.s. , and that, as a consequence, $\tilde{\rho}_M \sim \tilde{a}^{-3(1+w_M)}$. The expansion rate, $H_{\rm ST} \equiv d\log\tilde{a}/d\tilde{t}$, is given by

$$H_{\rm ST} = \frac{1+\alpha_M(\varphi)\,\varphi'}{A_M(\varphi)}\,\frac{\dot{a}}{a} \,, \tag{1.15}$$

where we have defined α_M according to eq. (1.10), and $(\cdot)' \equiv d(\cdot)/d\log a$. Using (1.15) and (1.14) in (1.8), we obtain the Friedmann equation in the ST theory,

$$H_{\rm ST}^2 = \frac{A_M^2(\varphi)}{3M_*^2}\frac{(1+\alpha_M(\varphi)\,\varphi')^2}{1-(\varphi')^2/6}\left[\tilde{\rho}_M + \tilde{V}\right] , \tag{1.16}$$

where $\tilde{V} \equiv A_M^{-4} V$. Comparing to eq. (1.1), we see that apart from the extra contribution to $\tilde{\rho}_{\rm tot}$ from the scalar field potential, the ST Friedmann equation differs from the standard one of GR by the presence of an effective, field-dependent Planck mass,

$$\frac{1}{3M_p^2} \to \frac{A_M^2(\varphi)}{3M_*^2}\frac{(1+\alpha_M(\varphi)\,\varphi')^2}{1-(\varphi')^2/6} \simeq \frac{A_M^2(\varphi)}{3M_*^2} \,, \tag{1.17}$$

where the last equality holds with very good approximations for all the choices of A_i functions considered in the present paper.

If the conformal factor $A_M^2(\varphi)$ is constant, then the full action $S_g + S_M$ is just that of GR (with $M_p = M_*/A_M$) plus a minimally coupled scalar field. Therefore, the coupling function α_M, defined according to eq. (1.10), measures the "distance" from GR of the ST theory, $\alpha_M = 0$ being the GR limit. Changing A_M, and, therefore, changing the effective Planck mass, opens the way to a modification of the standard relation between H and $\tilde{\rho}$, or T. In order to study the evolution of $A_M(\varphi)$, one should come back to eq. (1.9). Considering an initial epoch deeply inside radiation domination, we can neglect the contribution from the potential on the r.h.s. . The other contribution, the trace of the energy-momentum tensor $(\rho_M - 3\,p_M)$ is zero for fully relativistic components but turns on to positive values each time the temperature drops below the mass threshold of a particle in the thermal bath. Assuming a mass spectrum — e.g. that of the SM, or of the MSSM — one finds that this effect is effective enough to drive the scalar field evolution even in the radiation domination era [7].

The key point to notice is that if there is a field value, φ_0, such that $\alpha_M(\varphi_0) = 0$, this is a *fixed point* of the field evolution [8, 9]. Moreover, if α'_M is positive (negative) the fixed point is attractive (repulsive). Since $\alpha_M = 0$ corresponds to the GR limit, we see that GR is a — possibly attractive — fixed point configuration.

The impact on the DM relic abundance of a scenario based on this mechanism of attraction towards GR was considered in [7, 10]. Regardless of the particular form of the $A_M(\varphi)$ function, the requirement that an attractive fixed point towards GR exists implies that the effective Planck mass in the past was *not smaller* than today, that is to say that, at a certain temperature T, for instance at DM freeze out, the universe was expanding *not more slowly* than in the standard GR case. This is easy to understand, since the past values of φ, and then of $A_M(\varphi)$ are all such that

$$\log\frac{A_M(\varphi)}{A_M(\varphi_0)} = \int_{\varphi_0}^{\varphi} dx\,\alpha_M(x) > 0\,, \tag{1.18}$$

with φ between φ_0 and the next fixed point. Therefore, according to eq. (1.17), the ratio between $H_{\rm ST}$ and $H_{\rm GR}$,

$$\frac{H_{\rm ST}}{H_{\rm GR}} \simeq A_M^2(\varphi)\,, \tag{1.19}$$

can only decrease in time. Another way of seeing this, is by noticing that the r.h.s. of the field equation (1.9), is proportional to the field derivative of the effective potential $V_{\rm eff} = \rho_M + V$, where the field dependence of ρ_M is given by eq. (1.12). Then, neglecting again V, the field evolution will tend towards minimizing $A_M(\varphi)$ (if $w_M \leq 1/3$), therefore minimizing $H_{\rm ST}/H_{\rm GR}$.

Eq. (1.19) is obtained under the same approximation used in eq. (1.17), that is by neglecting the scalar field contribution to the total energy density, and assuming the same matter content in the ST and GR cases. Notice that these approximations are far from mandatory, and we only use them here in order to illustrate how the fixed point mechanism works. The numerical analysis we will present in the following were indeed obtained using

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the full expressions, such as eq. (1.16). Finally, in order to identify the fixed point with GR, we have to impose

$$\frac{1}{3M_p^2} = \frac{A_M^2(\varphi_0)}{3M_*^2}\,. \tag{1.20}$$

1.2 Lowering H in the past

Most of the the mechanisms discussed so far (i.e. adding relativistic d.o.f.'s, the kination scenario, or ST theories with a single matter sector) give a faster expansion of the universe in the past w.r.t. the standard case. In the case of ST theories with a single matter sector, we have just seen that this comes as a consequence of the requirement of an attractive fixed point towards GR. In this subsection, we will show that adding more matter sectors, with different conformal factors, allows us to keep the desirable property of late time convergence to GR and, at the same time, to have a lower expansion rate in the past. A discussion about mechanisms, different from ours, which can induce a slower expasion rate of the Universe has been given in ref. [11].

To illustrate our point for ST theories we will consider just two matter sectors, a "visible" one, containing the SM or one of its extensions, and a "hidden" one. The full action is then given by

$$S = S_g + S_v + S_h\,, \tag{1.21}$$

where the two matter actions S_v and S_h have two different conformal functions $A_v(\varphi)$ and $A_h(\varphi)$. The discussion follows quite closely that of the previous subsection. The only subtle point is to notice that, if $A_v(\varphi) \neq A_h(\varphi)$ there is no Weyl transformation that gives covariantly conserved energy-momentum tensors both for the visible and for the hidden sector. Since particle masses, reaction rates and so on, are computed in terms of parameters of the "visible" action, the transformation to perform in order to compare with the standard GR case is the one leading to a conserved $\tilde{T}^v_{\mu\nu}$, that is [4]

$$\tilde{g}_{\mu\nu} = A_v^2(\varphi) g_{\mu\nu}\,, \tag{1.22}$$

and so on. As a consequence, the expansion rate in this case is given by

$$H_{\rm ST}^2 = \frac{A_v^2(\varphi)}{3M_*^2}\frac{(1+\alpha_v(\varphi)\,\varphi')^2}{1-(\varphi')^2/6}\left[\tilde{\rho}_v + \tilde{\rho}_h + \tilde{V}\right]\,, \tag{1.23}$$

where

$$\tilde{\rho}_v \sim \tilde{a}^{-3(1+w_v)}\,,$$

while

$$\tilde{\rho}_h \sim \tilde{a}^{-3(1+w_h)}\left(\frac{A_h}{A_v}\right)^{1-3w_h}\,.$$

In order to study the existence of a fixed point, it is still convenient to revert to the Einstein Frame field equation, eq. (1.9). The r.h.s. , is now given by the field derivative of the effective potential

$$V_{\rm eff} = \rho_v + \rho_h + V\,, \tag{1.24}$$

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with the field-dependence of $\rho_{v,h}$ given by eq. (1.12). The condition to have a fixed point is then

$$\sum_{i=v,h} \alpha_i\,(1-3w_i)\rho_i + V' = 0\,, \tag{1.25}$$

while, asking that the fixed point is stable implies

$$\sum_{i=v,h} \left(\alpha_i'\,(1-3w_i)\rho_i + \alpha_i^2\,(1-3w_i)^2\rho_i + V''\right) \geq 0\,. \tag{1.26}$$

From eq. (1.23) we see that, away from the fixed point, $H_{\rm ST}$ is lower than the one obtained in GR with the same matter content but frozen scalar fields if

$$\frac{d^2}{d\varphi^2}\left(A_v^2(\varphi)\frac{1+\tilde{\rho}_h/\tilde{\rho}_v|_{\rm ST}}{1+\tilde{\rho}_h/\tilde{\rho}_v|_{\rm GR}}\right) < 0\,, \tag{1.27}$$

where, again, we have assumed that the second fraction in eq. (1.23) is approximately one, and we have neglected the contribution from the scalar potential.

As an example, we now consider A_i functions of the form

$$A_{v,h}(\varphi) = 1 + b_{v,h}\varphi^2\,. \tag{1.28}$$

Neglecting again the potential, we see that the fixed point condition, eq. (1.25), is solved by the symmetric point $\varphi = 0$. The stability condition, eq. (1.26), translates into

$$\sum_{i=v,h} b_i(1-3w_i)\rho_i \geq 0\,, \tag{1.29}$$

which, according to eq. (1.27), is compatible with a lower $H_{\rm ST}/H_{\rm GR}$ outside the fixed point (i.e. in the past), if

$$b_v < 0\,, \tag{1.30}$$

where we have assumed $\rho_h \ll \rho_v$ close to the fixed point, since we are interested in a physical situation in which most of the dark matter lives in the "visible" sector (as in the MSSM).

1.3 Numerical examples

To be implemented in a sensible cosmological model, the previously discussed mechanism for lowering the expansion rate in the past has to respect the severe bound coming from BBN, namely [12]

$$\frac{|H_{\rm ST}-H_{\rm GR}|}{H_{\rm ST}} < 10\,\% \qquad \text{at BBN}\,. \tag{1.31}$$

We now show in a few examples that indeed the bound (1.31) can be satisfied even by points of the parameters space $(b_v, b_h, \varphi_{\rm in})$ giving rise to a pre-BBN value of the ratio (1.19) as low as 10^{-3}. Such important deviations from standard cosmology are allowed in the present scenario by the effectiveness of the GR fixed point.

In order to numerically solve eqs. (1.7)–(1.9) we need the equation of state parameters

$$1-3w_i(y) = \frac{I_i(y)}{1+e^{y-y^i_{\rm eq}}} + \frac{1}{e^{y^i_{\rm eq}-y}+1} \qquad i = v, h \tag{1.32}$$

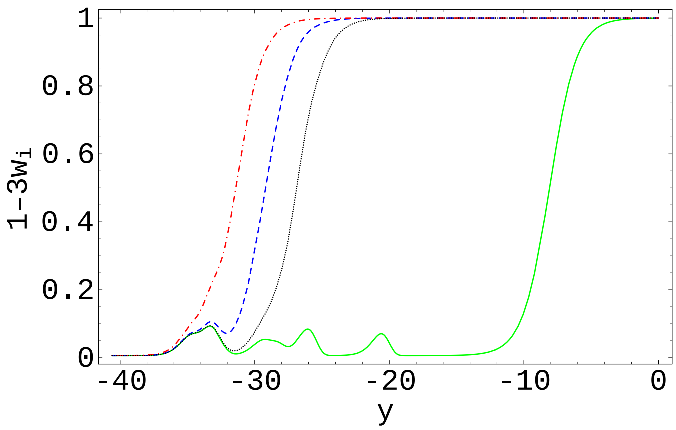

Figure 1: Four different combinations of the quantity $(1 - 3w_i)$ as a function of y. For all of them a MSSM-like mass spectrum is assumed (heaviest mass 1 TeV). The solid [green] line corresponds to the visible sector; the dot-dashed [red], dashed [blue] and dotted [black] lines represent a hidden sector with a hidden matter-hidden radiation equivalence at 10 GeV, 1 GeV and 0.1 GeV respectively.

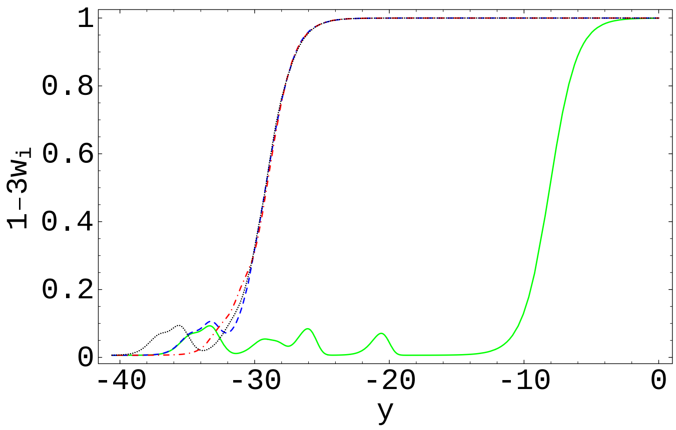

Figure 2: Four different combinations of the quantity $(1 - 3w_i)$ as a function of y. The solid [green] line corresponds to the visible sector with a MSSM-like mass spectrum (heaviest mass 1 TeV). The dot-dashed [red], dashed [blue] and dotted [black] lines represent a hidden sector with three different mass spectra: heaviest mass 0.1 TeV, 1 TeV and 10 TeV respectively. In the hidden sector the equivalence temperature is fixed at 1 GeV.

where $y = \log \tilde{a}$ and $y^i_{\rm eq}$ refers to the equivalence in the visible ($i = v$) and hidden ($i = h$) sectors respectively. The functions $I_i(y)$ are given by [7]

$$I_i(y) = \sum_{P_i} \frac{15}{\pi^2} \frac{g_{P_i}}{g_i} e^{2(y - y_{P_i})} \times \\ \times \int_0^{+\infty} \frac{z^2_{P_i} dz_{P_i}}{\sqrt{e^{2(y-y_{P_i})} + z^2_{P_i}} \left[e^{\sqrt{e^{2(y-y_{P_i})} + z^2_{P_i}}} \pm 1 \right]} \tag{1.33}$$

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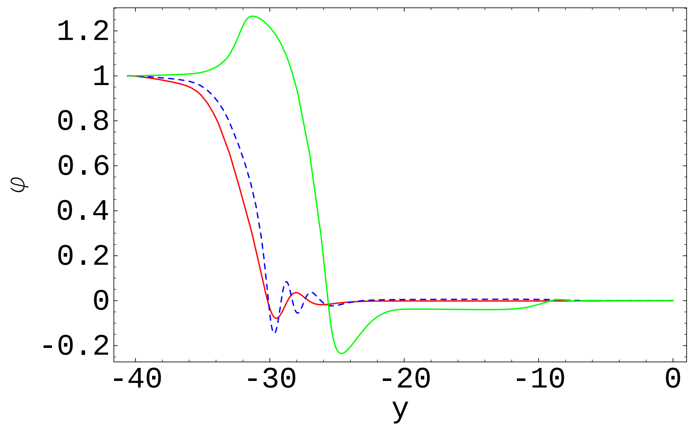

Figure 3: Evolution of the scalar field φ with y for three different choices of the parameters (b_v, b_h). The dark solid [red], dashed [blue] and light solid [green] lines correspond respectively to $(-0.2, 5)$ [Model 1], $(-0.4, 15)$[Model 2] and $(-0.521, 50)$ [Model 3]. BBN occurs at $y \simeq -22$.

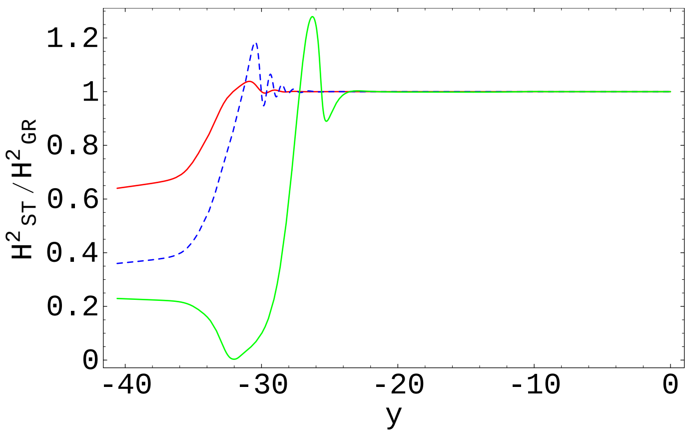

Figure 4: Ratio of the ST and GR Hubble rates squared $H^2_{\rm ST}/H^2_{\rm GR}$ as a function of y for three different choices of parameters (b_v, b_h). The dark solid [red], dashed [blue] and light solid [green] lines correspond respectively to $(-0.2, 5)$[Model 1] , $(-0.4, 15)$ [Model 2] and $(-0.521, 50)$ [Model 3]. BBN occurs at $y \simeq -22$.

where $y_{P_i} = -\log m_{P_i}/T_0$, m_{P_i} and g_{P_i} are the masses and the relativistic degrees of freedom of the particles P_i and $T_0 = 2.73$ K $\simeq 2.35 \times 10^{-13}$ GeV is the current temperature of the Universe.

As mass thresholds for the visible sector, namely y_{P_v}, we use the one given by a MSSM-like mass spectrum [1] while the equivalence time $y^v_{\rm eq}$ has been computed according to [13]. The analogous quantities for the hidden sector are free parameters of the theory that nevertheless do not have any drastic impact on the final result. The only significant assumption we do is that $\lim_{y\to\bar{y}}(1 - 3w_h(y)) \simeq 1$ for $\bar{y} \ll y_{\rm BBN}$. In such a way the contribution of the hidden sector to the r.h.s. of the scalar field equation before BBN can

[1]Only particles with a mass smaller than the temperature of the phase transition by means of which they become massive have to be considered (see also [7] and references therein).

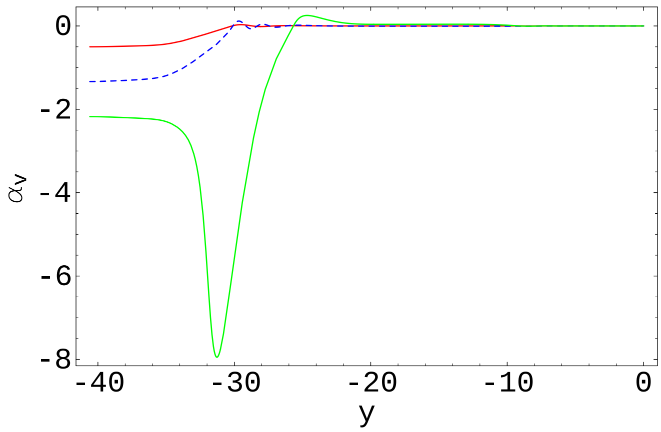

Figure 5: Evolution of the coupling function α_v with y for three different choices of parameters (b_v, b_h). The dark solid [red], dashed [blue] and light solid [green] lines correspond respectively to $(-0.2, 5)$ [Model 1], $(-0.4, 15)$ [Model 2] and $(-0.521, 50)$ [Model 3]. The BBN occurs at $y \simeq -22$

be dominant w.r.t. the one of the visible sector. The equation of state parameter w_h is plotted in figure (1) for three different values of $y^h_{\rm eq}$. In figure (2) instead, for a given value of $y^h_{\rm eq}$ we plot w_h for three different choices of mass thresholds in the hidden sector.

We can now integrate the equation of motion for the scalar field. Fixing as the initial condition $\varphi_{\rm in} = 1$ (in Planck units), we show in figure (3) the behavior of φ as a function of y for three different choices of parameters b_v and b_h. For the same choices we plot in figures (4) and (5) the evolution of the ratio $H^2_{\rm ST}/H^2_{\rm GR}$ and of the function α_v respectively. As anticipated, an agreement with the BBN bound is achieved even by solutions with a pre-BBN value of the ratio $H^2_{\rm ST}/H^2_{\rm GR}$ of order 10^{-3} (the minimal value in figure 4 for Model 3 is 0.003).

The scalar field dynamics can be qualitatively understood by having in mind the following rough estimation for its first derivative

$$\dot{\varphi} \propto -\sum_i \alpha_i (1 - 3w_i)\rho_i \,, \tag{1.34}$$

where each α_i is weighted by the the corresponding function $(1 - 3w_i)\rho_i$. If both α_i are positive, the scalar field is driven toward the fixed point $\phi = 0$. Analogously, negative values of the couplings α_i lead to a run-away behavior of φ. The present scenario corresponds to the case in which (before BBN) $\alpha_v < 0$ and $\alpha_h > 0$. Therefore, the interplay between the contributions of the hidden and visible sectors to the r.h.s. of eq. (1.34) becomes relevant. If at early time $(1 - 3w_h)\rho_h$ always dominates, then the only effect of having a $\alpha_v < 0$ is to realize the initial condition $H^2_{\rm ST}/H^2_{\rm GR} < 1$ (see Subsection 1.2). This is the case in Models 1 and 2 in figure (4). However, if at early time there is a short period where the visible sector contribution $(1 - 3w_v)\rho_v$ dominates, then the ratio $H^2_{\rm ST}/H^2_{\rm GR}$ decreases from its initial value until when $(1 - 3w_v) \simeq 1$ and, as a consequence, the hidden sector contribution becomes dominant. This happens in Model 3 of figure (4) where the parameter b_v has been tuned close to -1/2 [2].

[2]With an initial condition $\varphi_{\rm in} = 1$, smaller values of b_v would lead to a divergent value of α_v during the

Let us conclude this subsection with an estimation of the dependence of our results from the parameters b_v. According to [14], the level of fine-tuning Δ_λ on a parameter λ needed to get the required value of an observable $\mathcal{O}$ is given by $\Delta_\lambda = |(\lambda/\mathcal{O})(\partial\mathcal{O}/\partial\lambda)|$. Requiring at early time (when $\varphi \sim 1$) $\mathcal{O}= H^2_{\rm ST}/H^2_{\rm GR} \simeq 10^{-3}$ and choosing $\lambda = b_v \sim -1/2$ we find

$$\Delta_{b_v} \sim 2|(1+b_v)b_v|\,10^3 \sim 5\times 10^2\,. \tag{1.35}$$

Therefore, configurations with a small initial value of $H^2_{\rm ST}/H^2_{\rm GR}$ are very sensitive to the parameter b_v.

2. Effect on the particle dark matter relic abundance

A modification of the Hubble rate at early times has impact on the formation of dark matter as a thermal relic, if the particle freeze-out occurs during the period of modification of the expansion rate. ST cosmologies with a Hubble rate increased with respect to the GR case have been discussed in refs. [7, 10, 15], where the effect on the decoupling of a cold relic was discussed and bounds on the amount of increase of the Hubble rate prior to Big Bang Nucleosynthesis have been derived from the indirect detection signals of dark matter in our Galaxy. For cosmological models with an enhanced Hubble rate, the decoupling is anticipated, and the required amount of cold dark matter is obtained for larger annihilation cross-sections: this, in turn, translates into larger indirect detection rates, which depend directly on the annihilation process. In refs. [10, 15] we discussed how low-energy antiprotons and gamma-rays fluxes from the galactic center can pose limits on the admissible enchancement of the pre-BBN Hubble rate. We showed that these limits may be severe: for dark matter particles lighter than about a few hundred GeV antiprotons set the most important limits, which are quite strong for dark matter masses below 100 GeV. For heavier particles, gamma-rays are more instrumental in determining significant bounds. Further recent considerations on the effect of cosmologies with modified Hubble rate are discussed in refs. [16 – 23].

In the case of the cosmological models which predict a reduced Hubble rate, the situation is reversed: a smaller expansion rate implies that the cold relic particle remains in equilibrium for a longer time in the early Universe, and, as a consequence, its relic abundance turns out to be smaller than the one obtained in GR. In this case, the required amount of dark matter is obtained for smaller annihilation cross sections, and therefore indirect detection signals are depressed as compared to the standard GR case: as a consequence, no relevant bounds on the pre-BBN expansion rate can be set. On the other hand, for those particle physics models which typically predict large values for the relic abundance of the dark matter candidate, this class of ST cosmologies may have an important impact in the selection of the regions in parameter space which are cosmologically allowed.

In order to quantify the amount of reduction for the relic abundance of a dark matter candidate, we show in figure 6 the ratio between the neutralino relic abundance calculated in the ST and in the GR theories, as a function of the neutralino mass. In figure 6 we

evolution of φ.

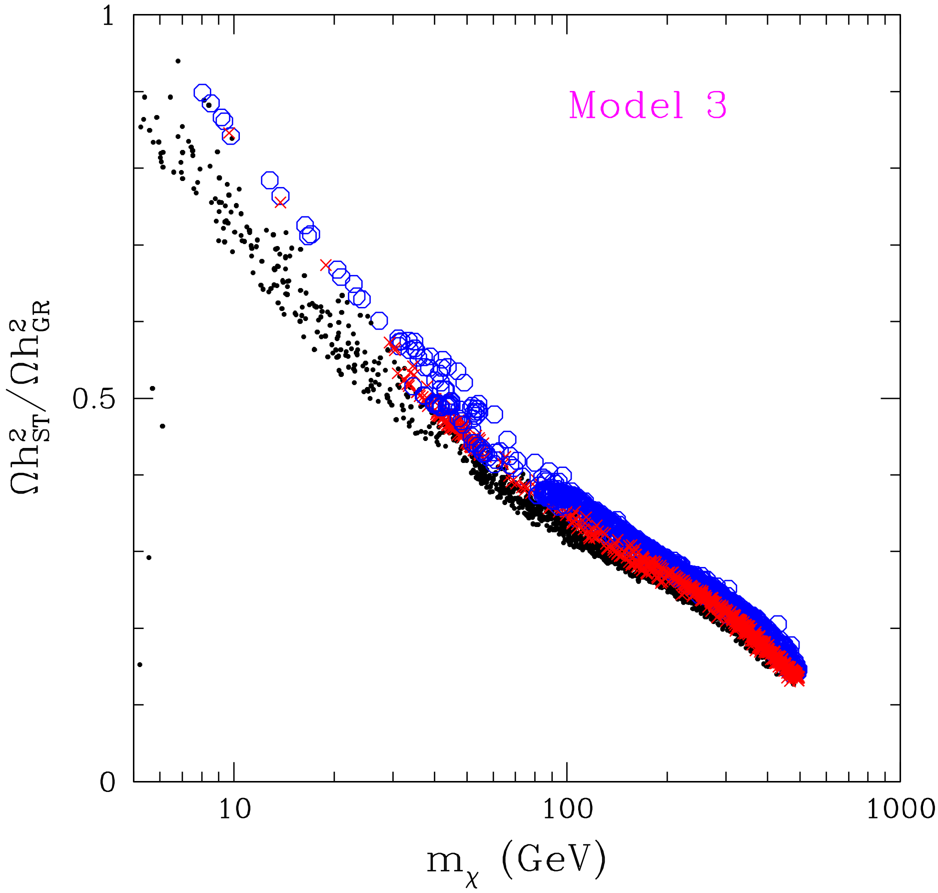

Figure 6: Ratio between the neutralino relic abundance calculated in the ST and in the GR theories, as a function of the neutralino mass, for the cosmological case of Model 3 of figure 4. Configurations with neutralino masses larger than 50 GeV refer to a scan of a low-energy realization of the MSSM [24], while those for lighter neutralinos refer to a gaugino non-universal MSSM [25 – 27]. The open [blue] circles refer to supersymmetric configurations where the GR relic abundance satisfies the dark matter upper bound of WMAP [13]. For the [red] crosses, the GR relic abundance is larger than the WMAP bound, but they are recovered in the ST case. For the [black] dots, the relic abundance is larger than the WMAP bound both for GR and ST (i.e. these are configurations excluded in both cosmologies).

have adopted the cosmological case of Model 3 of figure 4. Configurations with neutralino masses larger than 50 GeV refer to a scan of a low-energy realization of the MSSM [24], while those for lighter neutralinos refer to a gaugino non-universal MSSM [25 – 27]. The supersymmetric parameters of our models are: the U(1) and SU(2) gaugino masses M_1 and M_2, the Higgs mixing mass parameter μ, the ratio of the two Higgs v.e.v.'s $\tan\beta$, the mass of the CP-odd neutral Higgs boson m_A, the squark soft-mass $m_{\tilde{q}}$ common to all squarks, the slepton soft-mass $m_{\tilde{l}}$ common to all sleptons, a common dimensionless trilinear parameter A for the third family (the trilinear parameters for the other families being set equal to zero). These parameters have been varied in the following intervals: $1 \leq \tan\beta \leq 50$,

$100\,\text{GeV} \leq |\mu| \leq 1000\,\text{GeV}, 100\,\text{GeV} \leq M_2 \leq 1000\,\text{GeV}, 100\,\text{GeV} \leq m_{\tilde{q}}, m_{\tilde{l}} \leq 3000\,\text{GeV}$, $90\,\text{GeV} \leq m_A \leq 1000\,\text{GeV}$, $-3 \leq A \leq 3$ and M_1 is related to M_2 by the usual GUT-induced relation $M_1/M_2 = 5/3 \tan^2\theta_W \simeq 0.5$.

For the gaugino non-universal models, the parameters M_1 and M_2 are not related and their ratio defines a new free parameter $R = M_1/M_2$ which we take in the interval $0.005 \leq R \leq 0.5$. In this susy scheme, neutralinos lighter that in the standard MSSM are obtained (see refs [25 − 27] and references quoted teherein for details on their properties and phenomenology).

The following experimental constraints are imposed: accelerators data on supersymmetric and Higgs boson searches (CERN e^+e^- collider LEP2 [28] and Collider Detectors D0 and CDF at Fermilab [29]); measurements of the $b \to s + \gamma$ decay process [30]: 2.71 $\leq B(b \to s+\gamma)\cdot 10^{-4} \leq 4.39$ is employed here: this interval is larger than the experimental determination [30] in order to take into account theoretical uncertainties in the SUSY contributions [31] to the branching ratio of the process (for the Standard Model calculation, we employ the recent NNLO results from ref. [32]; see also discussion in the next Session for further details); the upper bound on the branching ratio $BR(B_s^0 \to \mu^- + \mu^+)$ [33]: we take $BR(B_s^0 \to \mu^- + \mu^+) < 1.2 \cdot 10^{-7}$; measurements of the muon anomalous magnetic moment $a_\mu \equiv (g_\mu - 2)/2$: for the deviation Δa_μ of the experimental world average from the theoretical evaluation within the Standard Model we use here the range $-98 \leq \Delta a_\mu \cdot 10^{11} \leq 565$, derived from the latest experimental [34] and theoretical [35] data. For the gaugino non-universal models, which entail light neutralinos, we also employ the bound arising from the invisible Z-width measurement for Z decaying into light neutralinos.

In figure 6 the open [blue] circles refer to supersymmetric configurations where the GR (standard cosmology) relic abundance satisfies the dark matter upper bound of WMAP [13], i.e. $\Omega h^2_{\rm GR} < 0.124$: these supersymmetric configurations are cosmologically viable in both cosmological models (standard GR and ST) but with a different contribution of neutralinos to the dark matter. For the [red] crosses, the GR relic abundance is larger than the WMAP bound, but these configurations are recovered in the ST case, since in this case $\Omega h^2_{\rm ST}$ is reduced enough with respect to the standard case, such that $\Omega h^2_{\rm ST} < 0.124$. For the [black] dots, the relic abundance is larger than the WMAP bound both for GR and ST and therefore these configurations are cosmologically excluded in both cosmologies.

For Model 3, we see that the amount of reduction in the neutralino relic abundance evolves as a function of the neutralino mass m_χ, ranging from 0.8-0.9 for $m_\chi \sim 10\,\text{GeV}$ to 0.1-0.2 for $m_\chi \sim 500\,\text{GeV}$. This dependence, which in figure 6 has been determined by numerically solving the relevant Boltzmann equation in both cosmologies, can be understood from the behaviour of the Hubble rate reduction shown in figure 4 and an approximate analytical solution for the cold dark matter relic abundance. By following and adapting the discussion of ref. [7], we analytically solve the Boltzmann equation:

$$\frac{dY}{dx} = -\frac{1}{x}\frac{s}{H}\langle\sigma_{\rm ann} v\rangle (Y^2 - Y^2_{\rm eq}) \tag{2.1}$$

in both cosmologies, i.e. by adopting the relevant Hubble rate H in GR or in ST. In eq. (2.1) Y is the relic particle density per comoving volume, $x = m/T$ with m the mass of

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the particle and T the temperature of the Universe, $s = (2\pi^2/45)\ h_\star(T)\ T^3$ is the entropy density (and $h_\star(T)$ denotes the effective number of degrees of freedom for the entropy) and $\langle\sigma_{\rm ann} v\rangle$ the particle annihilation cross-section. The solution of the Boltzmann equation can be formally obtained in the same way in both cosmologies, since we can define the Hubble rate H in eq. (2.1) as:

$$H^2 = A^2(T) H^2_{\rm GR} \tag{2.2}$$

where the temperature dependent function $A^2(T)$ is the one depicted in figure 4 and GR is recovered when $A(T) \to 1$. The same information can be translated in a change in the effective number of degrees of freedom $g_\star(T)$ at temperature T:

$$g_\star(T) \longrightarrow A^2(T) g_\star(T) \tag{2.3}$$

An approximated solution of eq. (2.1) can be cast in the standard form:

$$\frac{1}{Y_0} = \frac{1}{Y_f} + \sqrt{\frac{\pi}{45G}}\ m \int_{x_f}^{\infty} dx\ \frac{A^{-1}(x)\, G(x) \langle\sigma_{\rm ann} v\rangle}{x^2} \tag{2.4}$$

where $G(x) = h_\star(x)/g_\star^{1/2}(x)$ and Y_0 and Y_f are the particle abundances per comoving volume today and at freeze-out, respectively. The freeze-out temperature is obtained by solving the implicit equation:

$$x_f = \ln\left[0.038\, M_P\, g\, m\, \frac{\langle\sigma_{\rm ann} v\rangle_f\, x_f^{-1/2}}{A(x_f) g_\star^{1/2}(x_f)}\right] \tag{2.5}$$

For GR the function $A(T)$ is 1 and the standard formulae for the relic abundance and freeze-out temperature are recovered. In ST cosmologies with a reduce Hubble rate $A(x_f)$ is smaller than 1, and therefore the freeze-out temperature is shifted to lower temperatures of the Universe (x_f gets larger than in GR).

The particle relic abundance is then simply given by:

$$\Omega h^2 = \frac{m\, s_0\, Y_0}{\rho_{\rm crit}} \tag{2.6}$$

where s_0 is the present entropy density and $\rho_{\rm crit}$ denotes the critical density.

By employing standard approximations to the Boltzmann equation (2.1) [1, 7], we can determine the ratio of the relic abundance calculated in the ST theory as scompared to the GR case:

$$\mathcal{R} \equiv \frac{\Omega h^2_{\rm ST}}{\Omega h^2_{\rm GR}} \simeq A(x_f^{\rm ST})\, \frac{G(x_f^{\rm GR})}{G(x_f^{\rm ST})}\, \frac{x_f^{\rm ST}}{x_f^{\rm GR}} \tag{2.7}$$

Notice that the relic abundance reduction depends mostly on the value of the Hubble rate reduction function $A(x_f^{\rm ST})$, calculated at the freeze-out temperature (in ST cosmology). The ratio of the freeze-out temperatures in SR and GR cosmologies $x_f^{\rm ST}/x_f^{\rm GR}$ typically introduce an effect which can range from a few percent to tens of percent (up to a factor of 2 or so), while the ratio $G(x_f^{\rm GR})/G(x_f^{\rm ST})$ is typically very close to 1.

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By means of the approximate relation of eq. (2.7) we can now justify the behaviour of figure 6. Cold dark matter decouples, in standard cosmology, at a freeze-out temperatures of the order of 0.04-0.05 of their mass [1] ($x_f^{\rm GR} \sim$ 20-25). Light neutralinos, with masses around 10-20 GeV therefore decouple, in standard cosmology, at temperatures around 0.4-1 GeV, which in figure 4 corresponds to $-y \sim$ 27-28. In this temperature range, the Hubble rate in the ST cosmology is reduced by a factor of the order of 0.7-0.8. Neutralino decoupling is therefore delayed and the decoupling is shifted to sligthely smaller temperatures, as shown by eq. (2.5). The relic abundance reduction $\mathcal{R}$ is therefore close to 0.8-0.9, as determined with a more precise numerical analysis in figure 6.

For larger neutralino masses the effect is stronger, since the decoupling occurs at larger temperatures, where the Hubble rate is reduced more as compared to GR. For instance, for a mass of the order of 500 GeV, the decoupling in GR occurs at temperatures of the order of 20-25 GeV, which correspond in figure 4 to $-y \sim$ 32-33, i.e. to temperatures where the function $A(T)$ is close to its minimum. In this case the relic abunance in more depressed than in the previous example for light neutralinos and turns out to be reduced by a factor of the order of 0.1, as shown in figure 6. Since the minimum value for Model 3 in figure 4 is 0.003, the maximal reduction for the relic abundance can be of the order of $\sqrt{0.003} = 0.05$ (apart from a correction from $x_f^{\rm ST}/x_f^{\rm GR}$ in eq. (2.7)), for neutralinos which turn out to decouple when the function $A(T)$ is at its minimum.

In the case of the other two models presented in figure 4, which predict less prominent reductions for the Hubble rate in the region where neutralinos decouple ($-y \sim$ 27-34 for neutralino masses in the 10-1000 GeV range), the reduction of the relic abundance is consequently milder. From the behaviour of the curves in figure 4 and from the above discussions, we can conclude that the ST models we are analyzing predict reductions in the neutralino relic abundance fron values of $\mathcal{R}$ close to 1 up to $\mathcal{R}$ of the order of 0.05, depending on the particle mass and, to a lesser extent, annihilation cross section.

In conclusion of this section, we recall that classes of scenarios in which the relic density of dark matter is either reduced or enhanced have been discussed in the context of modified cosmologies other than the one presented in this paper. These classes contain models in which one has a late entropy production which dilutes the neutralino abundance [36 − 38], models in which the Universe at freeze-out is matter dominated [39], scenarios with non-thermal production [40, 41] or with a low-reheating temperature [42], which may also be coupled with a scalar field decay [43, 44], models of extra-dimensions and branes cosmology [16], string-cosmology [45], cosmic-strings decay [46] or inflaton decay [47, 48]. Analyses which discuss the impact on dark matter abundance and detection rates, in addition to refs. [10, 15], also been presented [49, 50].

3. Implications for dark matter in the CMSSM

A typical and noticeable case where the relic abundance constraint in GR cosmology is very strong is offered by minimal SUGRA models, where the neutralino is the dark matter candidate and its relic abundance easily turns out to be very large, in excess of the cosmological bound provided by WMAP [13]:

$$0.092 \leq \Omega_{\rm CDM} h^2 \leq 0.124 \tag{3.1}$$

Large sectors of the supersymmetric parameter space are excluded by this bound. A reduction of the expansion rate will therefore have a crucial impact on the allowed regions in parameter space, which are therefore enlarged or shifted. The potential reach of accelerators like the Large Hadron Collider (LHC) or the International Linear Collider (ILC) on the search of supersymmetry may therefore be affected by this broadening of the allowed parameter space, especially for the interesting situation of looking for supersymmetric configurations able to fully explain the dark matter problem.

We have therefore studied how the allowed parameter space of minimal SUGRA changes in ST cosmologies with a reduced Hubble rate. We have used a cosmological model of the type of Model 3 discussed in the previous section and depicted in figure 4, a cosmological model which represents a typical maximal impact of ST cosmologies with a reduced Hubble rate on the neutralino relic abundance. For the calculations of the neutralino relic density we have used here the DarkSUSYpackage [51], with an interface to ISAJET 7.69 [52] for the minimal SUGRA parameter space determination, with two major modifications. First, the relic density is obtained by the implementation of a numerical solution of a modified Boltzmann equation which includes the reduced Hubble rate evolution (similar to the method used in refs. [7, 10, 15] for the enhanced case and in the previous section for the MSSM). Second, the NNLO contributions to the Standard Model branching ratio of the $\mathrm{BR}[\bar{B} \to X_s\gamma]$ have been recently determined [32]: the updated result, which we use here, is $\mathrm{BR}[\bar{B} \to X_s\gamma]_{\rm SM} = (3.15 \pm 0.23) \times 10^{-4}$ ($E_\gamma > 1.6\,\mathrm{GeV}$). In order to implement the NNLO SM result with the supersymmetric contribution, which are known up to the NLO [53], we have used the following approximate expression, which is suitable when the beyond-standard-model (BSM) corrections are small [32, 54, 55]:

$$\begin{aligned}[\mathrm{BR}]_{\rm theory} \times 10^4 = \; & 3.15 \\ & - 8.0 \times \left(\delta_{\rm BSM}[C_7^{(0)}] + \frac{\alpha_s(\mu_0)}{4\pi} \delta_{\rm BSM}[C_7^{(1)}] \right) \\ & - 1.9 \times \left(\delta_{\rm BSM}[C_8^{(0)}] + \frac{\alpha_s(\mu_0)}{4\pi} \delta_{\rm BSM}[C_8^{(1)}] \right)\end{aligned} \tag{3.2}$$

where $C_i^{(j)}(\mu_0)$ are LO ($j = 0$) and NLO ($j = 1$) Wilson coefficients. For the matching scale μ_0 (which should be taken as $\mu_0 = 2M_W \sim 160\,\mathrm{GeV}$) we use $\mu_0 = \bar{m}_t(m_{t,\rm pole}) = 163.7\,\mathrm{GeV}$, and we use a top-quark pole mass $m_{t,\rm pole} = 171.4\,\mathrm{GeV}$ [56] and $\alpha_s(M_Z) = 0.1189$ [57]. The theoretical calculation in eq. (3.2) is compared to the current world average of the experimental determination [30]:

$$[\mathrm{BR}]_{\rm exp} \times 10^4 = \left(3.55 \pm 0.24 \begin{array}{c} +0.09 \\ -0.10 \end{array} \pm 0.03 \right) \tag{3.3}$$

The estimated error of the theoretical calculation of the Standard Model contribution is $\pm 0.23 \times 10^{-4}$ [32, 54]. For the theoretical beyond-SM correction we have assumed an error

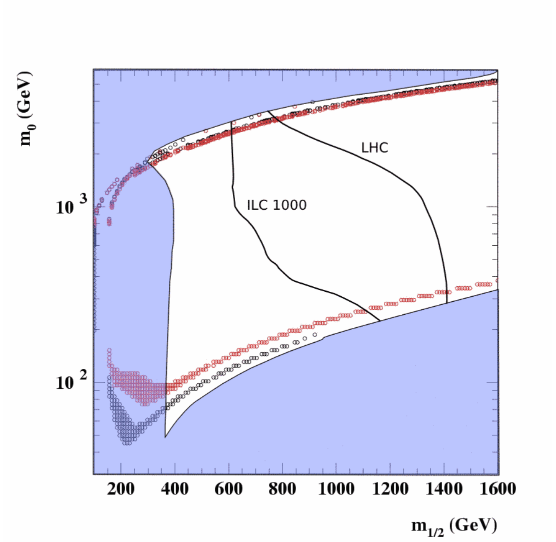

Figure 7: Regions in the $(m_{1/2}, m_0)$ parameter space where the neutralino relic abundance falls in the cosmological interval for cold dark matter obtained by WMAP, for $\tan\beta = 10$, $A_0 = 0$ and positive μ. In the bulk region, the lower [black] points refer to GR cosmology, while the upper [red] points stand for a ST cosmology with a reduced Hubble rate. The shaded areas are forbidden by theoretical arguments and experimental bounds. The two curves are indicative of the reach for $100\,\mathrm{fb}^{-1}$ of the LHC [61, 62] and of the ILC at $\sqrt{s} = 1000\,\mathrm{GeV}$ energy [61, 63].

of the same size. Adding all experimental and theoretical errors we get the following 2σ interval for the branching ratio:

$$2.71 \times 10^{-4} \leq \mathrm{BR}[\bar{B} \to X_s \gamma] \leq 4.39 \times 10^{-4} \tag{3.4}$$

Supersymmetric models for which the theoretical calculation in eq. (3.2) is outside this interval are considered in disagreement with the experimental result.

As a first example, we scan the universal gaugino mass $m_{1/2}$ and soft scalar-mass m_0 parameters of minimal SUGRA for a low value of the $\tan\beta$ parameter and a vanishing universal trilinear coupling A_0. The higgs-mixing parameter μ, derived by renormalization group equation (RGE) evolution and electro-weak symmetry breaking (EWSB) conditions, is taken as positive. Our choice of parameters is therefore here:

$$\tan\beta = 10 \qquad \mathrm{sgn}(\mu) = + \qquad A_0 = 0 \tag{3.5}$$

figure 7 shows the result in the plane $(m_{1/2}, m_0)$, both for the standard GR case and for the ST case of Model 3. Shaded areas denote regions which are excluded either by theoretical arguments or by experimental constraints on higgs and supersymmetry searches, as well as supersymmetric contributions to rare processes, namely to the $\mathrm{BR}[\bar{B} \to X_s \gamma]$ and to the muon anomalous magnetic moment $(g-2)_\mu$. More specifically, the upper wedge refers to the non-occurrence of the radiative EWSB and the lower-right area to the occurrence of a stau LSP. The low–$m_{1/2}$ vertical band is excluded by the quoted experimental bounds.

The sector of the supersymmetric parameter space which provides LSP neutralinos with a relic abundance in the cosmological range of eq. (3.1) are denoted by the open

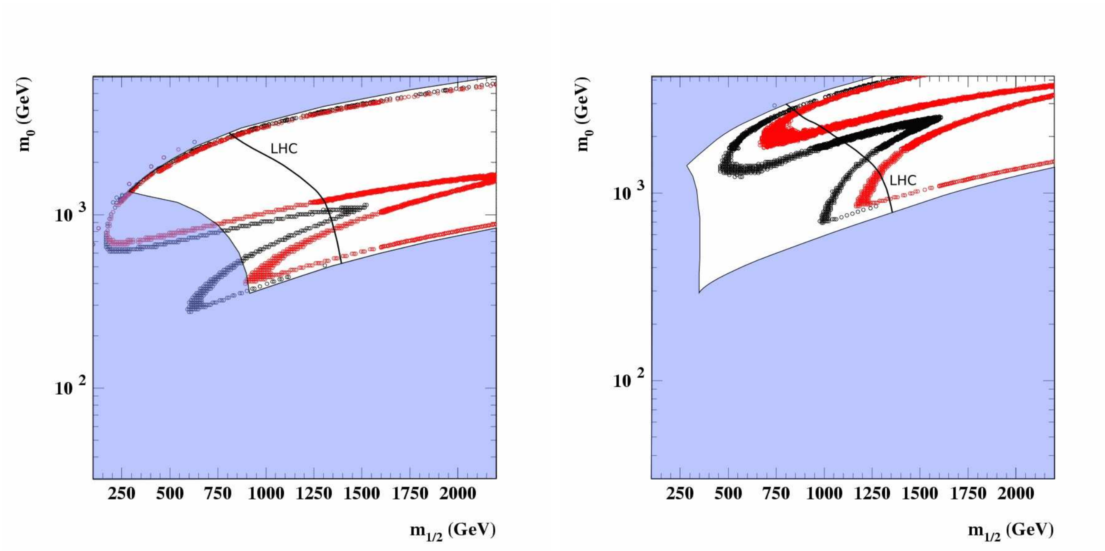

Figure 8: Regions in the $(m_{1/2}, m_0)$ parameter space where the neutralino relic abundance falls in the cosmological interval for cold dark matter obtained by WMAP. The left panel refers to $\tan\beta = 45$, $A_0 = 0$ and negative μ. The right panel is obtained for $\tan\beta = 53$, $A_0 = 0$ and positive μ. Notations are as in figure 7.

circles: in the so-called "bulk region" (low values of both $m_{1/2}$ and m_0), the lower [black] points fulfill the density constraint in the standard GR cosmology, while the upper [red] points in the modified ST cosmology with reduced Hubble rate. In the region above the points, the neutralino relic abundance exceeds the cosmological bounds, and therefore refers to supersymmetric configurations which are excluded by cosmology. figure 7 shows that in our modified cosmological scenario, the allowed regions in parameter space are enlarged (the relic density has been decreased 1.4 times to 4.4 times compared to the standard case) and those which refer to dominant neutralino dark matter are shifted towards larger values of the supersymmetric parameters $m_{1/2}$ and m_0. The bulk region now occurs for values of m_0 larger by a factor of 2 (while the bulk region for the GR case now refers to cosmologically subdominant neutralinos). Nevertheless, this sector of the parameter space is already mostly excluded by accelerator searches also in ST cosmologies. In the coannihilation channel, which extends for low values of the ratio $m_0/m_{1/2}$, along the boundary of the stau excluded region, the change is more dramatic: this coannihilation region, which appears to be fully explorable at the LHC, now extends towards larger values of $m_{1/2}$, beyond the estimated LHC reach.

In the cosmologically allowed region of large m_0, where a gaugino-mixing becomes possible and therefore the neutralino can efficiently annihilate and provide an acceptable relic abundance (a mechanism discussed in ref. [58] and lately dubbed as "focus point region" in ref. [59]), the effect of reducing the Hubble rate translates into a slight lowering of the cosmologically relevant region, with no drastic phenomenological effect in this case.

As a second example, we consider the case of large values of $\tan\beta$. We show two cases, one which refers to a negative μ parameter:

$$\tan\beta = 45 \qquad \text{sgn}(\mu) = - \qquad A_0 = 0 \tag{3.6}$$

and one to a positive value of μ:

$$\tan\beta = 53 \qquad \text{sgn}(\mu) = + \qquad A_0 = 0 \tag{3.7}$$

The results are shown in figure 8. In this case the change in the cosmological scenario is more relevant, not only in the coannihilation channel, but also in the "funnel"region [60] which occurs for intermediate values of the ratio $m_0/m_{1/2}$. In the GR case, almost the full cosmologically allowed parameter space may be explored by the LHC. When the ST cosmology is considered, the funnel region dramatically extends towards large values of m_0 and $m_{1/2}$ and goes well beyond the accelerators reach. Also the coannihilation region now extends to values of $m_{1/2}$ well in excess of 2 TeV. In the case of the positive μ reported in the right panel of figure 8, also the focus-point region shows a deviation from the GR case, and is shifted towards lower values of the m_0 parameter.

In all the examples shown above we see that the effect induced by a ST cosmology with a reduced Hubble rate on the cosmologically viable regions in the minimal SUGRA parameter space can be a sizeable shift of these regions toward larger values of the parameters m_0 and $M_{1/2}$. Regions which correspond to a neutralino relic abundance which is in the WMAP range for CDM (eq. (3.1)) are shifted as shown in figures 7, 8. As a consequence, regions which are cosmologically acceptable, since the refer to $\Omega h^2 < 0.124$ (and therefore include also subdominant neutralinos) are enlarged.

A fraction, in same cases sizeable, of the cosmologically viable parameter space in ST cosmologies fall beyond the reach of LHC, and will therefore not be directly tested at current accelerators. On the other hand, there are large portions of the parameter space, which correspond to sectors of the theory for which LHC has the capabilities to reconstruct at least partially the susy spectrum and couplings, which correspond to susy configurations which are *not* cosmologically viable in GR cosmology. If it will happen that LHC discovers compatibility with supersymmetry in such regions in minimal SUGRA, the compatibility with cosmology could then be recovered by cosmological models which predict reduced expansion rate, and the models discussed in this paper show an explicit realization in the framework of ST theories.

If supersymmetry is discovered at the LHC, the possibility to desentangle GR from ST cosmologies is nevertheless not a simple task (as well as for any other modification of the standard cosmological model). Celarly if LHC would discover supersymmetry compatible with a sector of parameter space where neutralinos represent the dominant dark matter component in GR, this would be the most economic and logical chioice (even though the same susy sector is compatible with the ST cosmology). If, on the other hand, LHC would discover susy in a sector which is hardly incompatible with neutralino dark matter (like, for intance, in a sector of parameter space around $m_{1/2} \sim 750$ GeV and $m_0 \sim 2$ TeV in the right panel of figure 8), this will point toward the necessity to accept a modified cosmology not to spoil the cosmological predictions of the susy model. ST theories with a reduced Hubble rate could then be a viable possibility.

Obviously it will not be easy (and is some cases even not feasible) to shape down allowed regions of parameter space with enough precision to clearly desentanlge between the GR and the ST domains. For this reaon, experiments like the ILC will be very helpful to refine the determination of susy parameters, once susy is discovered.

Let us also comment that a handle on the possibility to desentangle between GR and ST if susy will be discovered at LHC could be offered by the indirect detection of dark matter: since in out ST cosmology the relic abundance is lower than in GR, a neutralino which is compatible with the WMAP interval in GR is typically subdmoninat in ST. This implies that that neutralino represents the whole galactic dark matter in GR, but just a fraction in ST, and therefore they constribute to galactic dark only fractionally and proportionally to their relic abundance [64 – 67].

In this case, neutralino dark matter annihilation would produce indirect detection signals (gamma-rays [68, 66], positrons [69, 70], antiprotons [71], antideuterium [72, 73]) at a lower rate in ST as compared to GR. This feature could be exploited to try to determine whether cosmology is standard or instead it requires a reduced Hubble rate, and if this reduction is compatible to the predictions of our ST cosmologies.

In figures 7, 8 we have considered the cosmological model which produces the largest effect (Model 3). As discussed in the previous section, different ST cosmologies predict smaller reductions of the relic abundance and therefore they imply smaller shifts in the cosmologically acceptable regions. We can therefore consider the results shown in figures 7, 8 as representative of the maximal effect for our class of cosmological models.

4. Conclusions

We have discussed Scalar Tensor cosmologies by determining under what conditions these theories can predict an expansion rate which is reduced as compared to the standard General Relativity case. We showed that in the case of ST theories with a single matter sector, the typical behaviour is an enchancement of the Hubble rate in the past: this arises as a consequence of the requirement of an attractive fixed point towards GR at late times. Instead, when additional matter sectors, with different conformal factors, are added, we can mantain the desirable property of late time convergence to GR and, at the same time, obtain a reduced expansion rate in the past. We showed that, for suitable choices of the parameters which govern the scalar field evolution, a sizeable reduction (up to about 2 orders of magnitude) of the Hubble rate prior to Big Bag Nucleosynthesis can be obtained. Large reductions come along with some fine-tuning on the scalar field parameters, while a milder decrease occurs without tuning problems.

We have then applied the results obtained on the reduction of the early-time Hubble rate to the formation of dark matter and the determination of its relic abundance. If the dark matter decouples during the period of Hubble-rate reduction, the relic abundance turns out to be reduced as compared to the standard GR case. We have shown that the effect typically is larger for larger neutralino masses ranging from a factor of 0.8-0.9 for masses of the order of 10 GeV to a factor of 0.1-0.2 for masses of the order of 500 GeV, for neutralinos in various low-energy realization of the MSSM.

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The reduction effect can be sizeable and can therefore have an important impact on the determination of the cosmologically allowed parameter space in minimal SUGRA models, where, in large portions of the parameter space and for the GR case, the neutralino relic abundance is large and in excess of the WMAP bound. We have therefore explicitely shown what are the modifications to the minimal SUGRA allowed parameter space when ST cosmologies with a reduced Hubble rate are considered and we have quantified the effect in view of the reach of LHC and ILC on the searches for supersymmetry at future accelerators. These modifications move the cosmologically relevant regions up to a few TeV for the $m_{1/2}$ parameters, since they significantly extend the coannihilation corridor and the funnel region which occurs at large values of $\tan\beta$.

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