



# One-loop weak corrections to $\gamma/Z$ hadro-production at finite transverse momentum<sup>☆</sup>

Ezio Maina<sup>a,b</sup>, Stefano Moretti<sup>c</sup>, Douglas A. Ross<sup>c</sup>

<sup>a</sup> *Dipartimento di Fisica Teorica, Università di Torino, Italy*

<sup>b</sup> *Istituto Nazionale di Fisica Nucleare, Sezione di Torino, Via Pietro Giuria 1, 10125 Torino, Italy*

<sup>c</sup> *School of Physics and Astronomy, University of Southampton, Highfield, Southampton SO17 1BJ, UK*

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## Abstract

We illustrate the effects of one-loop weak corrections onto the production of neutral gauge bosons of the Standard Model at RHIC-spin, Tevatron and LHC, in presence of quark/gluon radiation from the initial state. We find such effects to be rather large, up to  $\mathcal{O}(10\text{--}20\%)$  in typical observables at all such colliders, where the cross section is measurable, thus advocating their inclusion in precision analyses.

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## 1. Prompt-photon and on-shell $Z$ hadro-production

The neutral-current processes ( $V = \gamma, Z$ )

$$q\bar{q} \rightarrow gV \quad \text{and} \quad q(\bar{q})g \rightarrow q(\bar{q})V \quad (1)$$

with  $V \rightarrow \ell^+\ell^-$  are two of the cleanest probes of the partonic content of (anti)protons, in particular of antiquark and gluon densities. In order to measure the

latter it is necessary to study the vector boson  $p_T$  spectrum. According to [1,2] the gluon density dominates for  $p_T > Q/2$  where  $Q$  is the lepton pair invariant mass. In the presence of polarised beams these reactions give access to the spin-dependent gluon distribution which is presently only poorly known. Thanks to the introduction of improved algorithms [3–5] for the selection of (prompt) photons generated in the hard scatterings (1), as opposed to those generated in the fragmentation of the accompanying gluon/quark jet, and to the high experimental resolution achievable in reconstructing  $Z \rightarrow \ell^+\ell^-$  ( $\ell = e, \mu$ ) decays, they are regarded—together with the twin charged-current channels

$$q\bar{q}' \rightarrow gW \quad \text{and} \quad q(\bar{q})g \rightarrow q'(\bar{q}')W, \quad (2)$$

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*E-mail address:* [stefano@hep.phys.soton.ac.uk](mailto:stefano@hep.phys.soton.ac.uk) (S. Moretti).

wherein  $W \rightarrow \ell\nu_\ell$ —as precision observables in hadronic physics. In fact, in some instances, accuracies of order one percent are expected to be attained in measuring these processes [6], both at present and future proton–(anti)proton experiments. These include the Relativistic Heavy Ion Collider running with polarised proton beams (RHIC-spin) at BNL ( $\sqrt{s}_{pp} = 300\text{--}600$  GeV), the Tevatron collider at FNAL (Run 2,  $\sqrt{s}_{p\bar{p}} = 2$  TeV) and the Large Hadron Collider (LHC) at CERN ( $\sqrt{s}_{pp} = 14$  TeV).

Not surprisingly then, a lot of effort has been spent over the years in computing higher order corrections to all such Drell–Yan type processes. To stay with the neutral-current ones, the subject of this paper, these include next-to-leading order (NLO) QCD calculations of both prompt-photon [7,8] and vector boson production [9]. QCD corrections to the  $p_T$  distributions have been computed in Refs. [10,11]. As for the full  $\mathcal{O}(\alpha)$  electro-weak (EW) corrections to  $Z$  production and continuum neutral-current processes (at zero transverse momentum), these have been completed in [12] (see also [13]), building on the calculation of the QED part in [14].

In the case of polarised (anti)proton beams, the process of calculating higher order corrections has proceeded more slowly [15,16]. NLO QCD corrections to the transverse momentum spectrum of Drell–Yan type processes via neutral-currents in presence of (longitudinal) spin effects from the initial state can be found for the non-singlet case in [17,18], while the complete calculation has been recently published in Ref. [19] (see also Ref. [20]).

The relatively large impact of one-loop EW corrections, as compared to the QCD ones, can be understood (see Refs. [21,22] and references therein for reviews) in terms of the so-called Sudakov (leading) logarithms of the form  $\alpha_W \log^2(\hat{s}/M_W^2)$ , which appear in the presence of higher order weak corrections (hereafter,  $\alpha_W \equiv \alpha_{EM}/\sin^2\theta_W$ , with  $\alpha_{EM}$  the electromagnetic (EM) coupling constant and  $\theta_W$  the weak mixing angle).<sup>1</sup> These ‘double logs’ are due to a lack of cancellation of infrared (both soft and collinear) virtual and real emission in higher order contributions

due to  $W$ -exchange in spontaneously broken non-Abelian theories.

The problem is, in principle, present also in QCD. In practice, however, it has no observable consequences, because of the averaging on the colour degrees of freedom of partons, forced by their confinement into colourless hadrons. This does not occur in the EW case, where, e.g., the initial state can have a non-Abelian charge, dictated by the given collider beam configuration. Modulo the effects of the parton distribution functions (PDFs), which spoil the subtle cancellations among subprocesses with opposite non-Abelian charge, for example, this argument holds for an initial quark doublet in proton–(anti)proton scatterings. These logarithmic corrections (unless the EW process is mass-suppressed) are universal (i.e., process independent) and are finite (unlike in QCD), as the masses of the EW gauge bosons provide a physical cut-off for  $W$ -boson emission. Hence, for typical experimental resolutions, softly and collinearly emitted weak bosons need not be included in the production cross section and one can restrict oneself to the calculation of weak effects originating from virtual corrections. In fact, one should recall that real weak bosons are unstable and decay into high transverse momentum leptons and/or jets, which are normally captured by the detectors. In the definition of an exclusive cross section then, one tends to remove events with such additional particles. Under such circumstances, the (virtual) exchange of  $Z$ -bosons also generates similar logarithmic corrections,  $\alpha_W \log^2(\hat{s}/M_Z^2)$ . Besides, the genuinely weak contributions can be isolated in a gauge-invariant manner from purely EM effects, at least in some simpler cases—which do include processes (1) but not (2)—and the latter may or may not be included in the calculation, depending on the observable being studied.

A further aspect that should be recalled is that weak corrections naturally introduce parity-violating effects in observables, detectable through asymmetries in the cross section, which are often regarded as an indication of physics beyond the Standard Model (SM) [6,23,24]. These effects are further enhanced if polarisation of the incoming beams is exploited, such as at RHIC-spin [25,26]. Comparison of theoretical predictions involving parity-violation with experimental data is thus used as another powerful tool for confirming or disproving the existence of some beyond the SM sce-

<sup>1</sup> In some cases, leading ( $\sim \alpha_W^n \log^{2n}(s/M_W^2)$ ), sub-leading ( $\sim \alpha_W^n \log^{2n-1}(s/M_W^2)$ ) and sub-sub-leading ( $\sim \alpha_W^n \times \log^{2n-2}(s/M_W^2)$ ) logarithms can be resummed.

narios, such as those involving right-handed weak currents [27], contact interactions [28] and/or new massive gauge bosons [29,30].

In view of all this, it becomes of crucial importance to assess the quantitative relevance of weak corrections affecting processes (1), (2). It is the aim of our Letter to report on the computation of the full one-loop weak effects entering processes (1) while the study of those for (2) will be deferred to a future publication [31].

## 2. Calculation and results

Since we are considering weak corrections that may be identified via their induced parity-violating effects and since we wish to apply our results to the case of polarised proton beams, it is convenient to work in terms of helicity matrix elements (MEs). Here, we define the helicity amplitudes by using the formalism discussed in Ref. [32]. At one-loop level such helicity amplitudes acquire higher order corrections from: (i) self-energy insertions on the fermions and gauge bosons; (ii) vertex corrections and (iii) box diagrams. All such contributions are pictured in Fig. 1.

The self-energy and vertex correction graphs contain ultraviolet divergences that have been subtracted here by using the ‘modified’ minimal subtraction ( $\overline{\text{MS}}$ ) scheme at the scale  $\mu = M_Z$ . Thus the couplings are taken to be those relevant for such a subtraction: e.g., the EM coupling,  $\alpha_{\text{EM}}$ , has been taken to be  $1/128$  at the above subtraction point. The one exception to this renormalisation scheme has been the case of the self-energy insertions on external fermion lines, which have been subtracted on mass-shell, so that the external fermion fields create or destroy particle states with the correct normalisation.

All these graphs are infrared and collinear convergent so that they may be expressed in terms of Passarino–Veltman [33] functions which are then evaluated numerically. The expressions for each of these diagrams have been calculated using FORM [34] and checked by an independent program based on FeynCalc [35]. For the numerical evaluation of the scalar integrals we have relied on the FORTRAN package FF [36]. A further check on our results has been carried out by setting the polarisation vector of the  $V$ -boson proportional to its momentum and verifying that

the sum of all one-loop diagrams vanishes, as required by gauge and BRST invariance. The full expressions for the contributions from these graphs are too lengthy to be reproduced here.

In both processes in (1), external (anti)quarks have been taken massless and both vector bosons ( $V = \gamma, Z$ ) have been put on-shell. In contrast, the top quark entering the loops in both reactions has been assumed to have the mass  $m_t = 175$  GeV. The  $Z$  mass used was  $M_Z = 91.19$  GeV and was related to the  $W$ -mass,  $M_W$ , via the SM formula  $M_W = M_Z \cos \theta_W$ , where  $\sin^2 \theta_W = 0.232$ . (Corresponding widths were  $\Gamma_Z = 2.5$  GeV and  $\Gamma_W = 2.08$  GeV.) For the strong coupling constant,  $\alpha_s$ , we have used the one-loop expression at  $Q^2 = \hat{s}$  with  $\Lambda_{\overline{\text{MS}}}^{(n_f=4)}$  chosen to match the value required by the (LO) PDFs used. The latter were Gehrman–Stirling set A for RHIC and Martin–Roberts–Stirling–Thorne set 2001 LO for Tevatron/LHC [37].

### 2.1. RHIC

The following beam asymmetries can, e.g., be defined at RHIC-spin:

$$\begin{aligned} A_{LL} d\sigma &\equiv d\sigma_{++} - d\sigma_{+-} + d\sigma_{--} - d\sigma_{-+}, \\ A_L d\sigma &\equiv d\sigma_- - d\sigma_+, \\ A_{PV} d\sigma &\equiv d\sigma_{--} - d\sigma_{++}. \end{aligned} \quad (3)$$

The first two are parity-conserving while the last two are parity-violating. Figs. 2, 3 show the NLO distributions in such quantities for both  $\gamma$  and  $Z$  final states, alongside those for the total cross section, as a function of the transverse momentum  $p_T$ , within the pseudo-rapidity range  $|y| < 1$ . The corrections due to full one-loop weak effects are also presented (for the cases in which the Born level result is non-zero). Effects onto the total cross sections are rather small, below the percent level, as expected, because of the rather low centre-of-mass (CM) energy available at partonic level, which is comparable with  $M_W$  and  $M_Z$ , so that logarithmic corrections are not enhanced. Nonetheless, in the case of  $\gamma$ -production, at  $p_T = 10$  GeV, the total NLO yearly rates of approximately 150 000 and 1 350 000 events accessible at low and high energy, respectively, contain a sizable contribution due to purely weak effects, about 200 and 1500 events in correspondence of  $\sqrt{s}_{pp} = 300$  and 600 GeV (for the

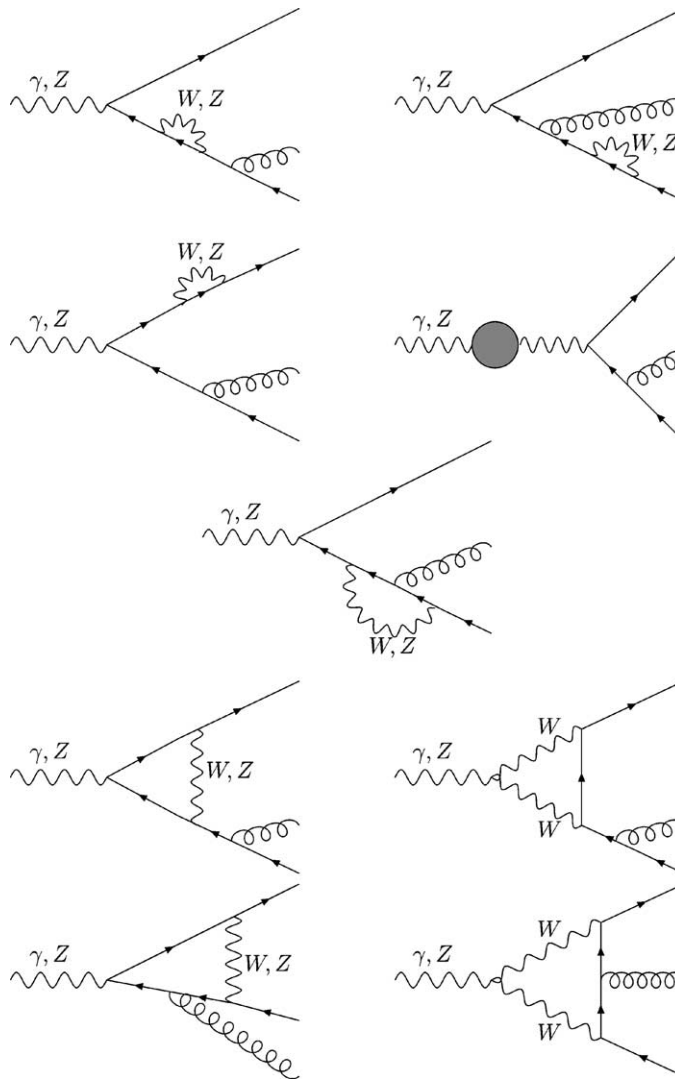


Fig. 1. Graphs describing processes (1) in the presence of one-loop weak corrections. The shaded blob represents all the contributions to the gauge boson self-energy and is dependent on the Higgs mass (we have set  $M_H = 115$  GeV). We neglect loops involving the Higgs boson coupling to the fermion line. The graphs in which the exchanged gauge boson is a  $W$ -boson are accompanied by those in which the latter is replaced by its corresponding Goldstone boson. There is a similar set of diagrams in which the direction of the fermion line is reversed, with the exception of the last graph, as here reversal does not lead to a distinct topology.

values of luminosity 200 and 800  $\text{pb}^{-1}$ , respectively). In case of  $Z$ -production, only at 600 GeV NLO effects are sizable, as they are responsible for 3 events being subtracted (the correction is negative) to the LO prediction of 457 events (at  $p_T = 10$  GeV). (The LO rate at 300 GeV for  $Z$ -production at such a transverse momentum is of only 7 events, unaffected by NLO weak effects.)

Rather large effects do appear in general for the asymmetries, particularly for the case of  $Z$  boson final states. In fact, for the latter, in the case of  $A_L$  and  $A_{LL}$ , they can range up to  $\pm(15\text{--}20)\%$ , while they are somewhat lower for the case of  $A_{PV}$ , 5% or so. Unfortunately, none of such NLO effects on the asymmetries is detectable, because of the poor production rate of  $Z$ -bosons. For photonic final states,

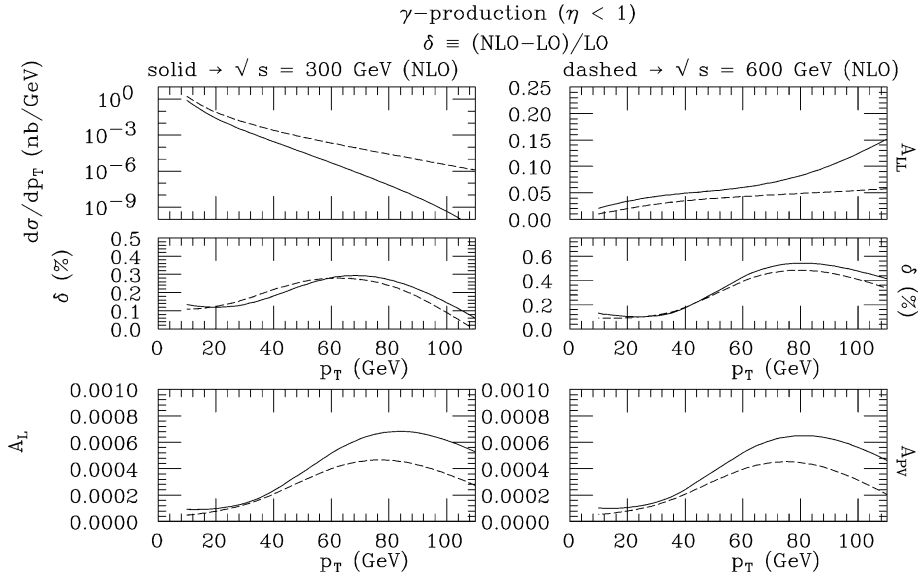


Fig. 2. The transverse momentum dependence of the  $\gamma$ -boson cross section in (1) as well as of the beam asymmetries in (3) at NLO (large frames) and the size of the one-loop weak corrections (small frames, limitedly to case in which the latter is non-zero), at RHIC-spin ( $\sqrt{s}_{pp} = 300$  and  $600$  GeV). Notice that the pseudorapidity range of either particle in the final state is limited to  $|\eta| < 1$ .

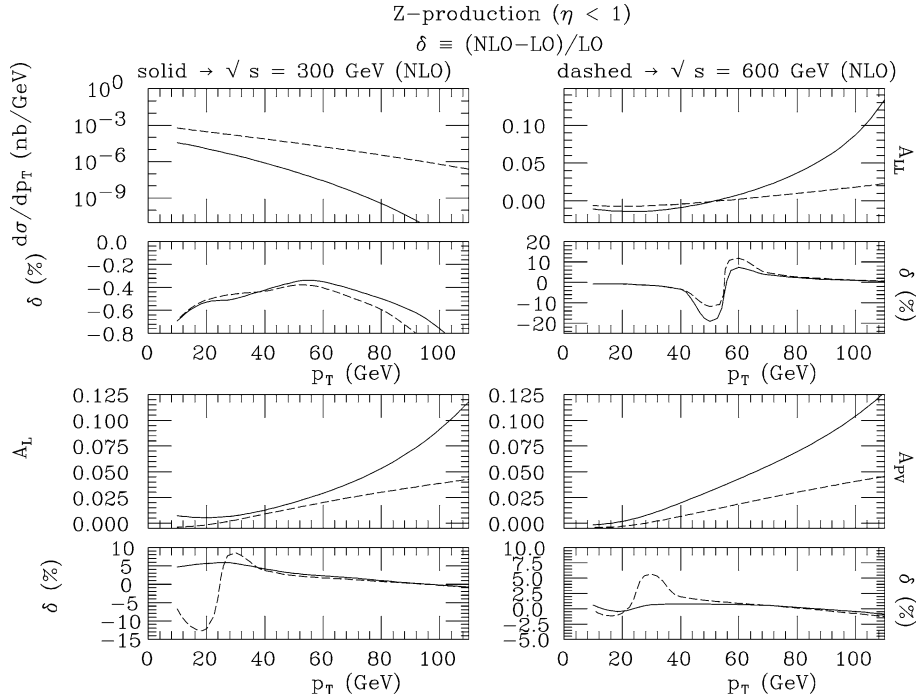


Fig. 3. The transverse momentum dependence of the  $Z$ -boson cross section in (1) as well as of the beam asymmetries in (3) at NLO (large frames) and the size of the one-loop weak corrections (small frames), at RHIC-spin ( $\sqrt{s}_{pp} = 300$  and  $600$  GeV). Notice that the pseudorapidity range of either particle in the final state is limited to  $|\eta| < 1$ .

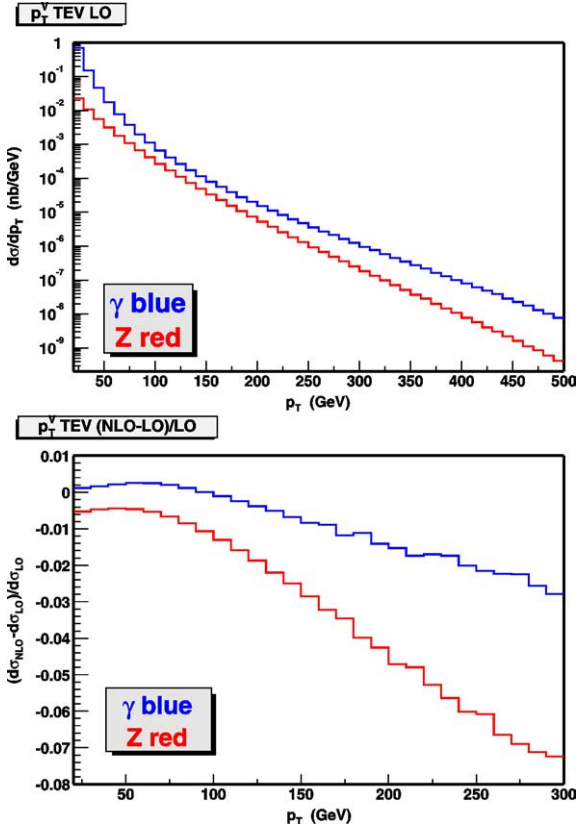


Fig. 4. The transverse momentum dependence of the  $\gamma$ - and Z-boson cross sections in (1) at LO (top frame) and the size of the one-loop weak corrections (bottom frame), at Tevatron ( $\sqrt{s_{pp}} = 2$  TeV). Notice that the pseudorapidity range of the jet in the final state is limited to  $|\eta| < 3$ .

one-loop weak effects on  $A_{LL}$  are not much larger than those on the total rates, nonetheless, they might just be observable at low  $p_T$ . For the cases of  $A_L$  and  $A_{PV}$ , which for the photon are exactly zero at Born level in massless QCD, one-loop weak effects are too poor to be observed experimentally.

## 2.2. Tevatron and LHC

Figs. 4, 5 show the effects of the  $\mathcal{O}(\alpha_S\alpha_{EW}^2)$  terms relatively to the  $\mathcal{O}(\alpha_S\alpha_{EW})$  Born results ( $\alpha_{EM}$  replaces  $\alpha_{EW}$  for photons), as well as the absolute magnitude of the latter, as a function of the transverse momentum, at Tevatron and LHC, respectively. The corrections are found to be rather large at both colliders, particularly

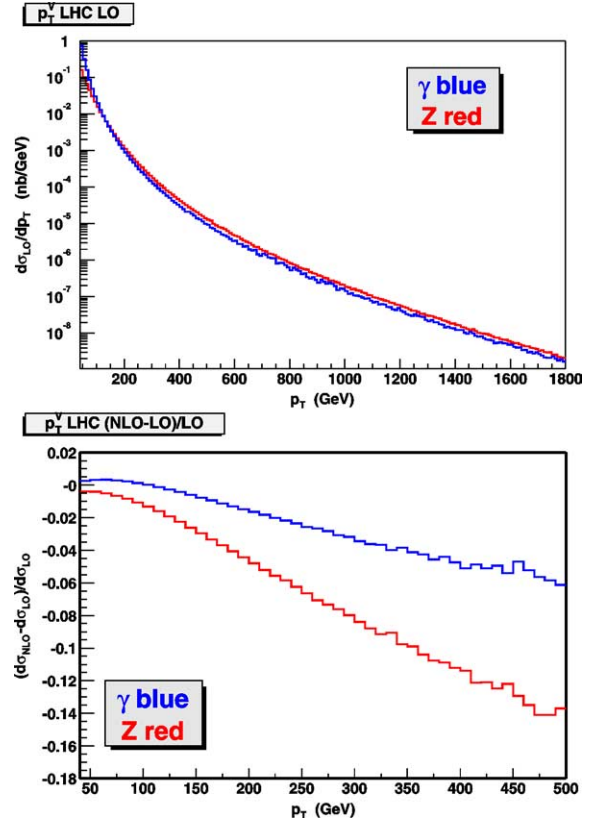


Fig. 5. The transverse momentum dependence of the  $\gamma$ - and Z-boson cross sections in (1) at LO (top frame) and the size of the one-loop weak corrections (bottom frame), at LHC ( $\sqrt{s_{pp}} = 14$  TeV). Notice that the pseudorapidity range of the jet in the final state is limited to  $|\eta| < 4.5$ .

for Z-production. In case of the latter, such effects are of order  $-7\%$  at Tevatron for  $p_T \approx 300$  GeV and  $-14\%$  at LHC for  $p_T \approx 500$  GeV. In general, above  $p_T \approx 100$  GeV, they tend to (negatively) increase, more or less linearly, with  $p_T$ . Such effects will be hard to observe at Tevatron but will indeed be observable at LHC. For example, at FNAL, for Z-production and decay into electrons and muons with  $BR(Z \rightarrow e, \mu) \approx 6.5\%$ , assuming  $L = 2\text{--}20 \text{ fb}^{-1}$  as integrated luminosity, in a window of 10 GeV at  $p_T = 100$  GeV, one finds 500–5000 Z + j events at LO, hence a  $\delta\sigma/\sigma \approx -1.2\%$  EW NLO correction corresponds to only 6–60 fewer events. At CERN, for the same production and decay channel, assuming now  $L = 30 \text{ fb}^{-1}$ , in a window of 40 GeV at  $p_T =$

450 GeV, we expect about 2000  $Z + j$  events from LO, so that a  $\delta\sigma/\sigma \approx -12\%$  EW NLO correction corresponds to 240 fewer events. In line with the normalisations seen in the top frames of Figs. 4, 5 and the size of the corrections in the bottom ones, absolute rates for the photon are similar to those for the massive gauge boson while  $\mathcal{O}(\alpha_S\alpha_{EW}^2)$  corrections are about a factor of two smaller.

### 3. Conclusions

Altogether, the results presented here point to the relevance of one-loop  $\mathcal{O}(\alpha_S\alpha_W^2)$  weak contributions for precision analyses of prompt-photon and neutral Drell–Yan events at both Tevatron and LHC, also recalling that the residual scale dependence of the known higher order QCD corrections to processes of the type (1) is very small in comparison [11]. Another relevant aspect is that such higher order weak terms introduce parity-violating effects in hadronic observables [26], which might just be observable at (polarised) RHIC-spin, particularly in the case of photons. (The case for polarised beams at the LHC, enabling the study of parity-violating asymmetries on the same footing as at RHIC-spin, is currently being discussed as one of the possible upgrades of the CERN collider.) The size of the mentioned corrections, relative to the lowest order results, is insensitive to the choice of PDFs. EM effects were neglected here because they are not subject to logarithmic enhancement, thus smaller with respect to the weak ones, or parity-violating effects either. However, their computation is currently in progress [31].

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