A network approach to investigate the aggregation phenomena in sports

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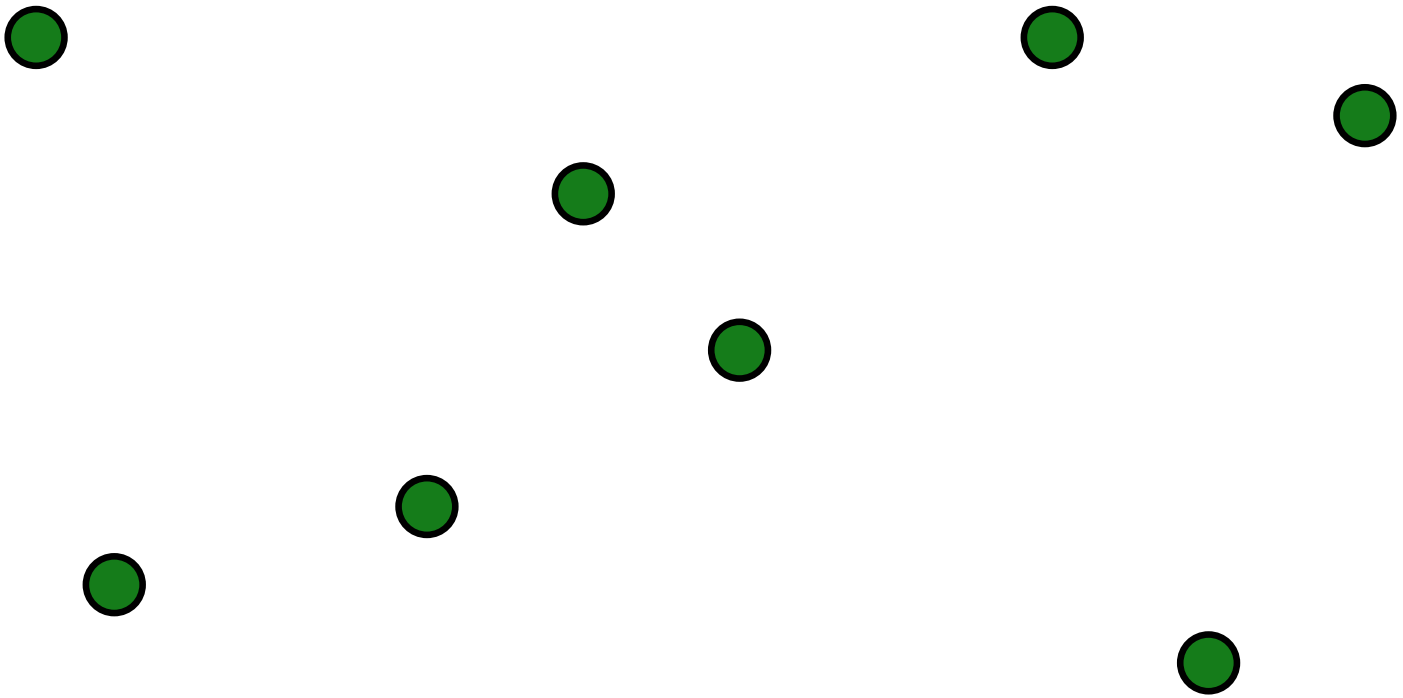
University of Lausanne, Switzerland

CS-SPORTS
Paris 12th August 2011

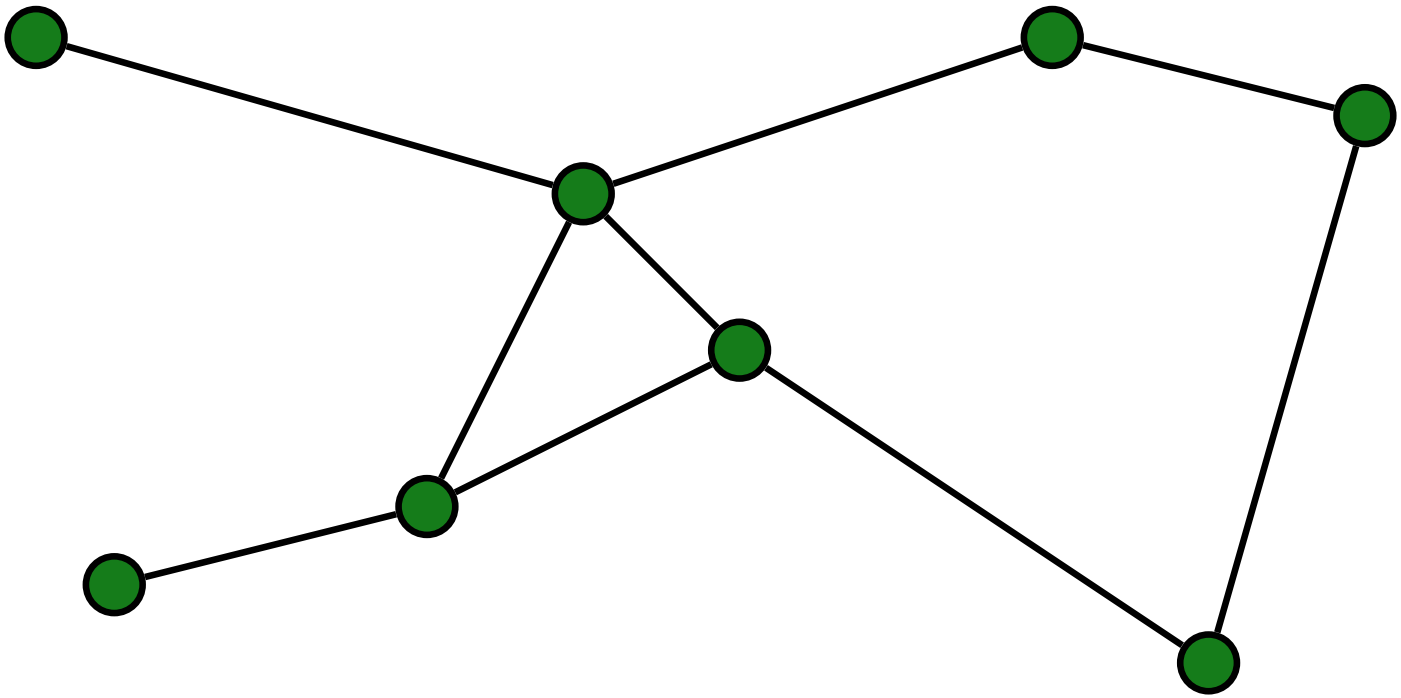
Network: definition

A network is given by a set of nodes

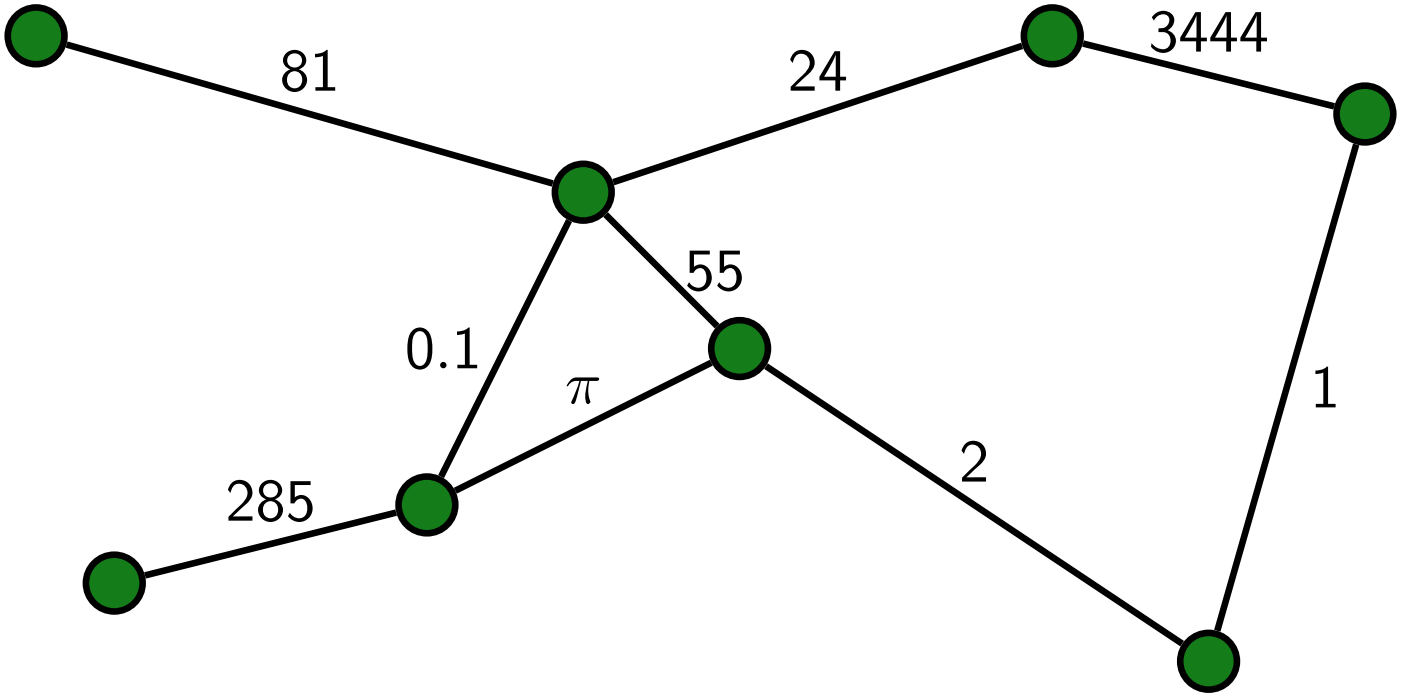


Network: definition

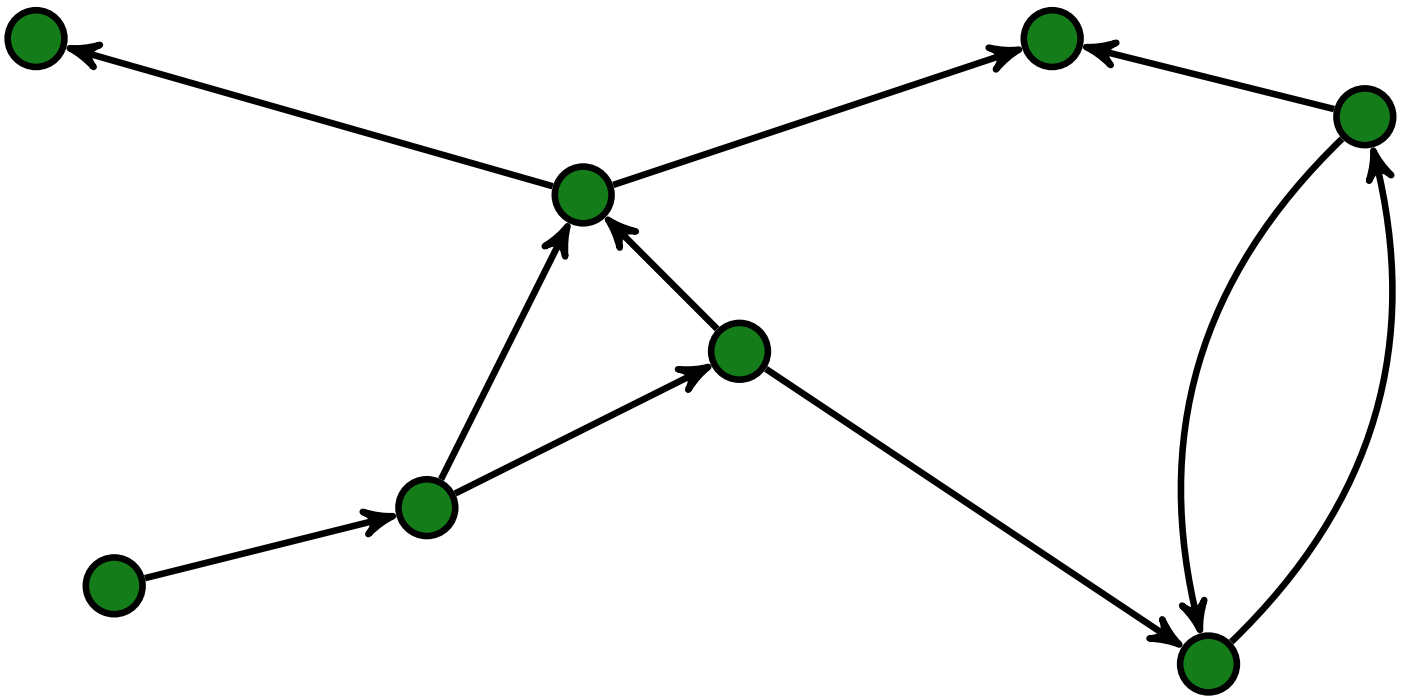
A network is given by a set of nodes and of interactions among nodes called edges



Weighted Networks



Directed Networks



Complex Networks: Nodes Centralities

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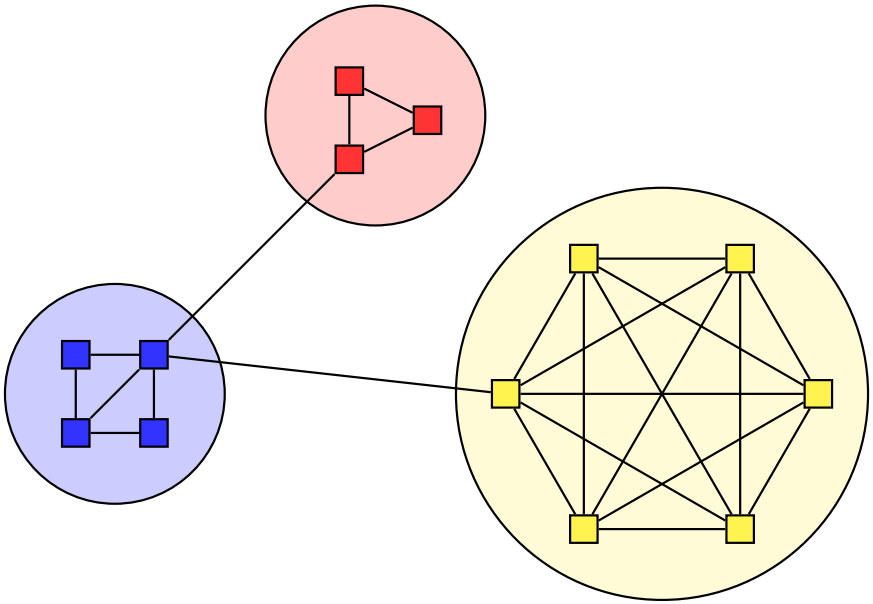
- degree-centrality: the importance of a node grows proportionally with its degree
- betweenness-centrality: the importance of a node given by the number of paths of minimum lenght that cross the node
- eigenvector-centrality: the importance of a node is proportional to the sum of the importance of all vertices that point to it (Newman 2003):

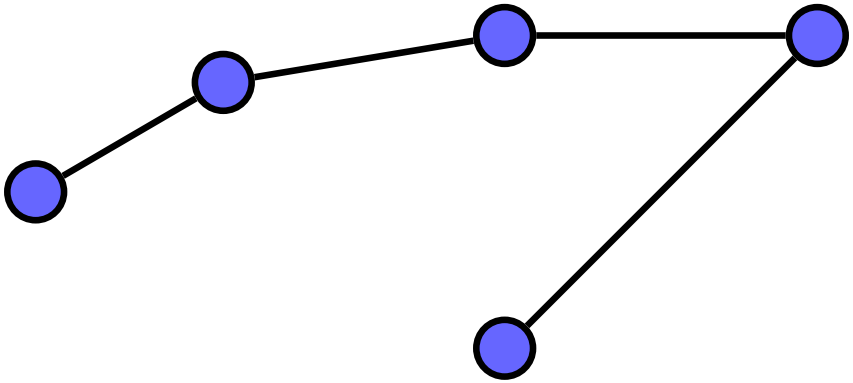
Complex Networks: Communities Detection

A community is defined as a subnet having few number of edges departing from it



Complex Networks: Distance among Nodes

Distance among nodes is defined as the minimum number of edges necessary to connect two nodes.



The shortest path in network is called the radius of the network while the longest is the diameter. In real world network it has been observed the small-world phenomena: a small diameter compared with the number of nodes.

Complex Networks: Assortativity

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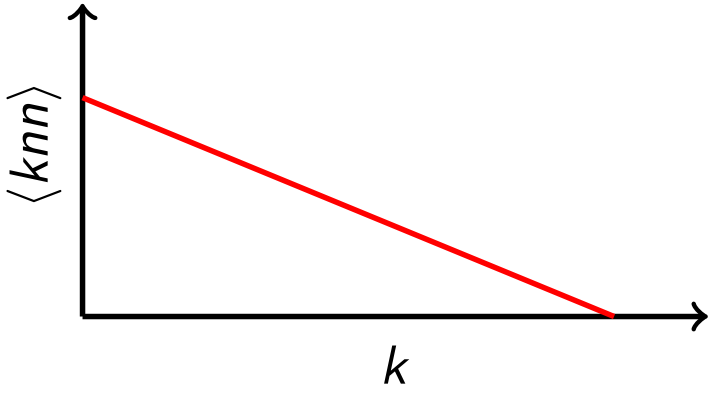

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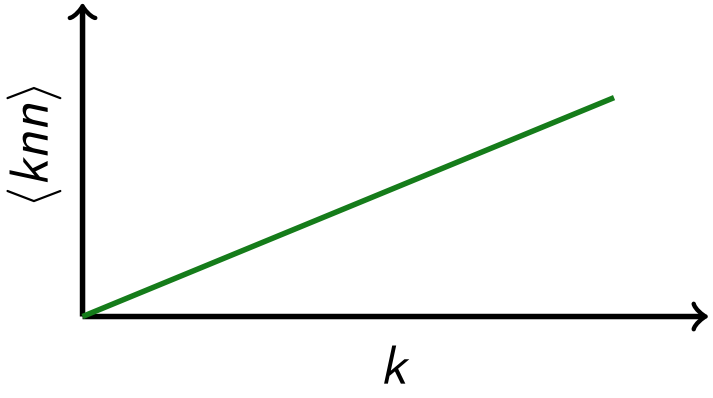
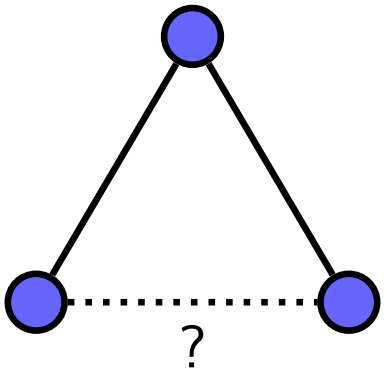


Complex Networks: Clustering Coefficient

The clustering coefficient of a node is a measure of how its neighbors are connected.



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Complex networks could be distinct in:

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a measure of the heterogeneity is given by:

$$\frac{\langle k \rangle}{\langle k^2 \rangle}$$

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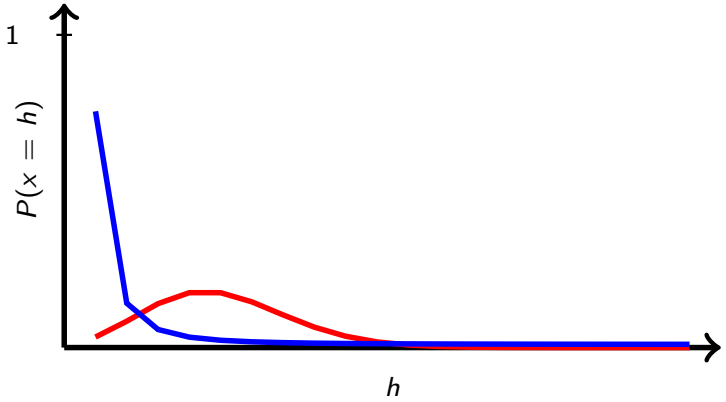
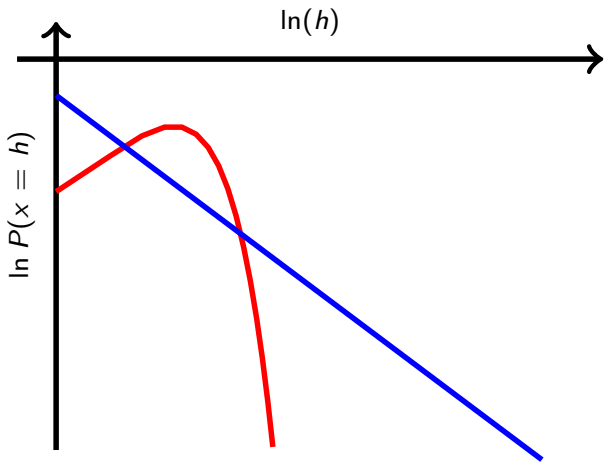
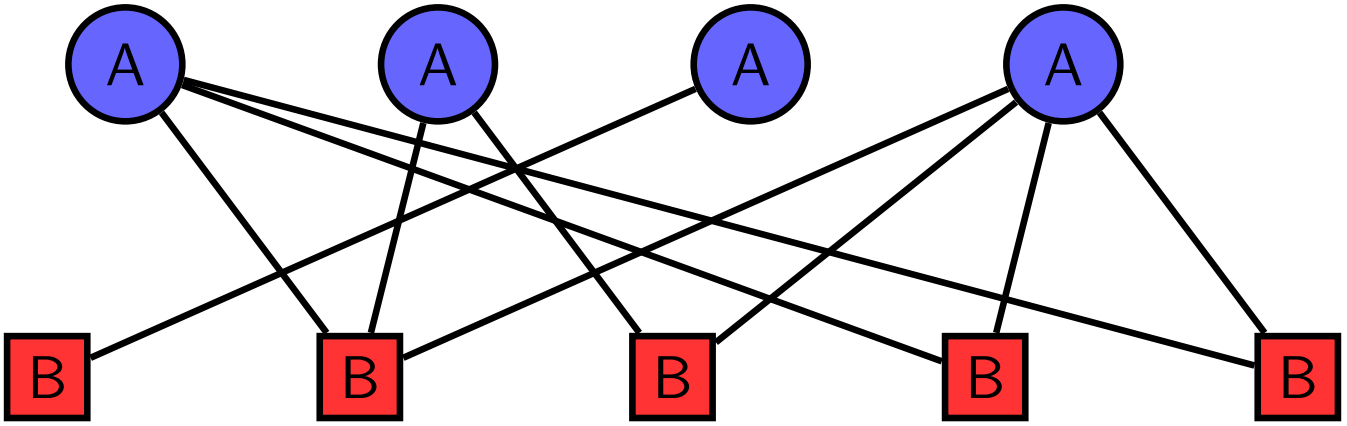




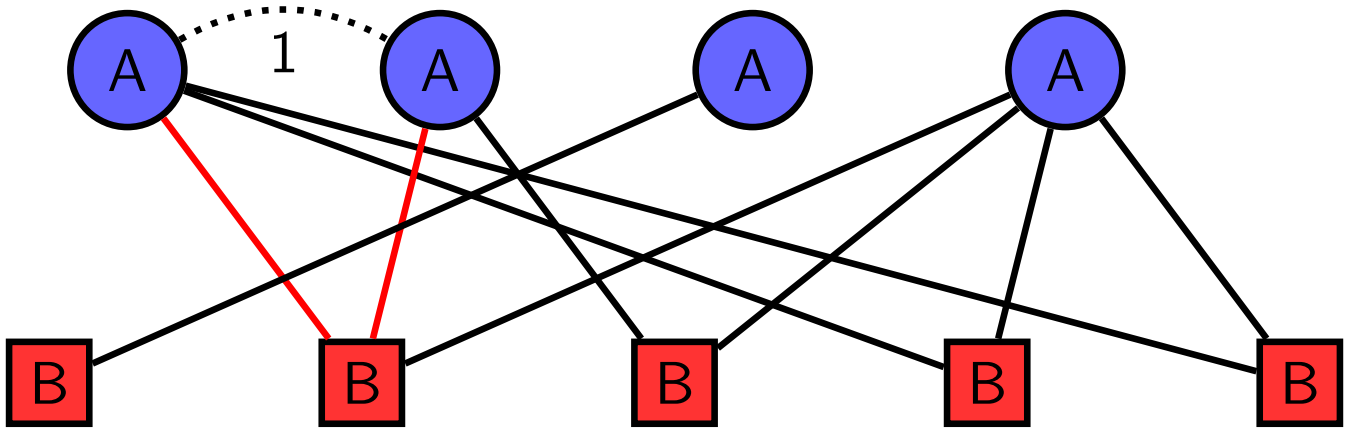
Bipartite Networks



Many example:

- co-authorship network;
- diseasome;
- heterosexual contact network;
- vector-borne disease network;

Bipartite Networks



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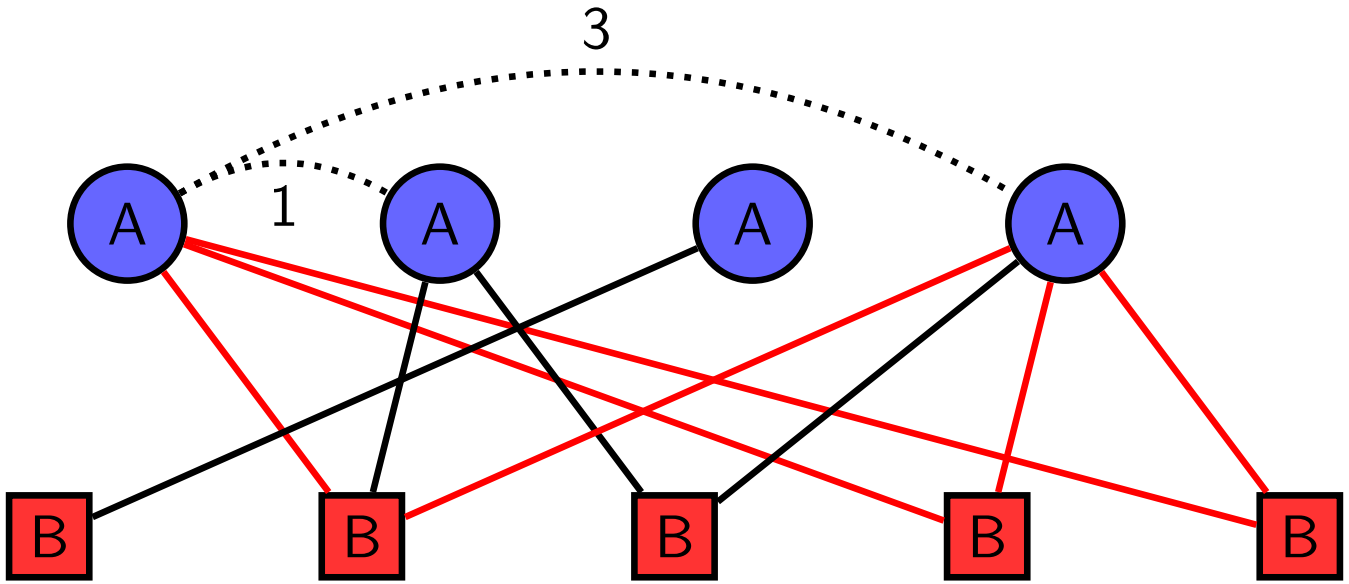

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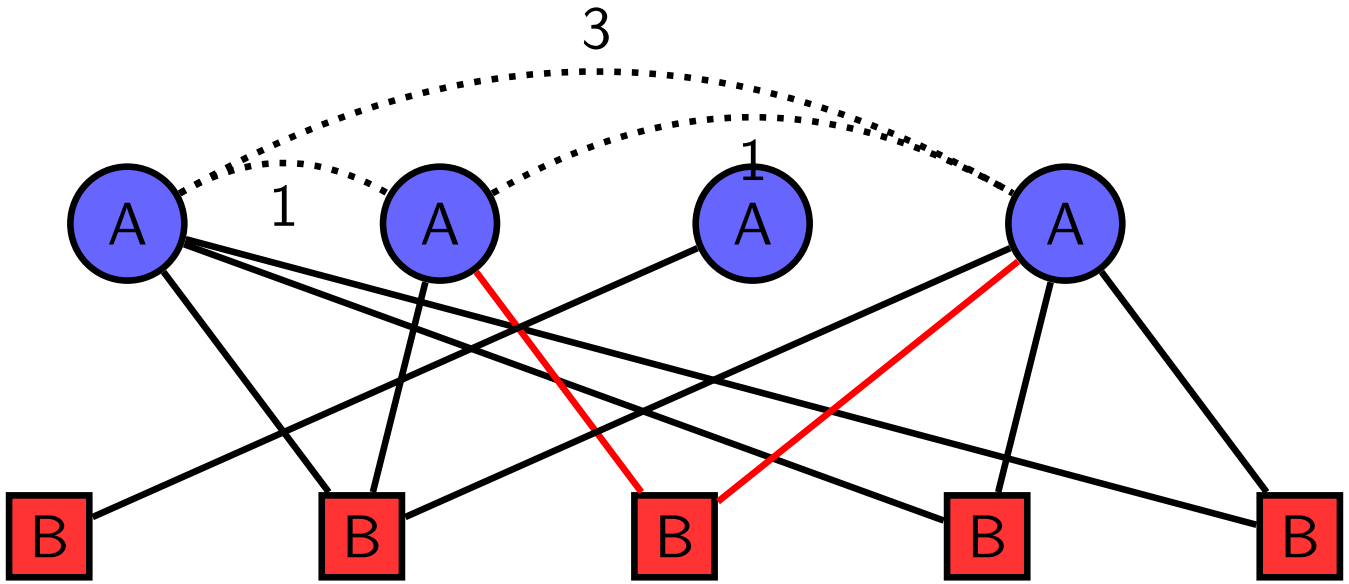
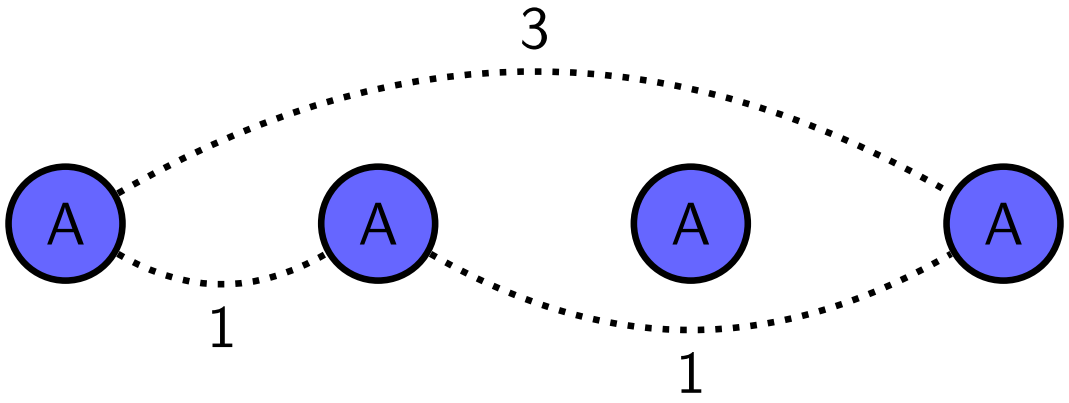
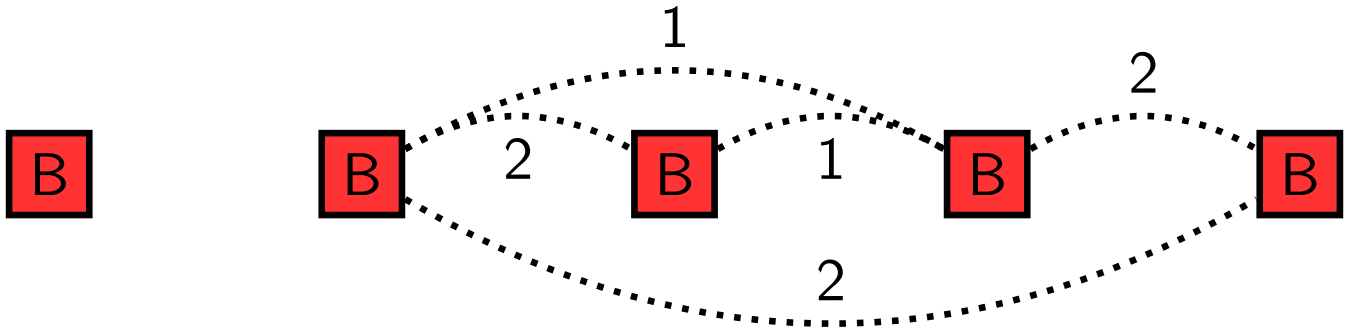


Bipartite Networks

The A-projection



the B-projection



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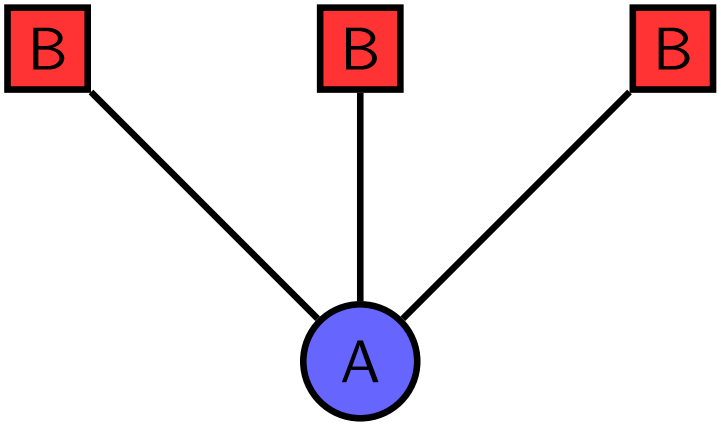

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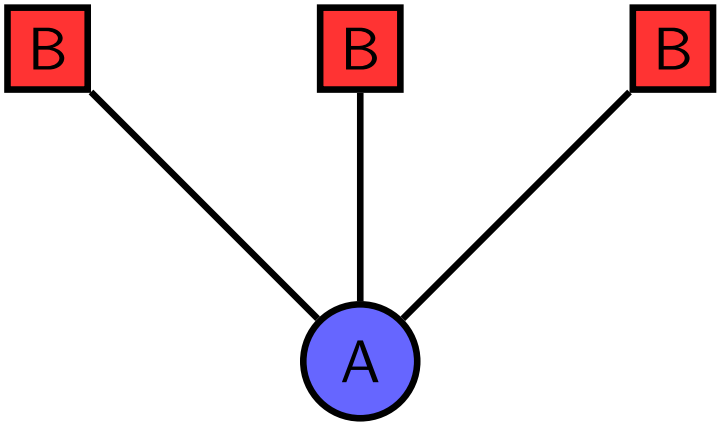

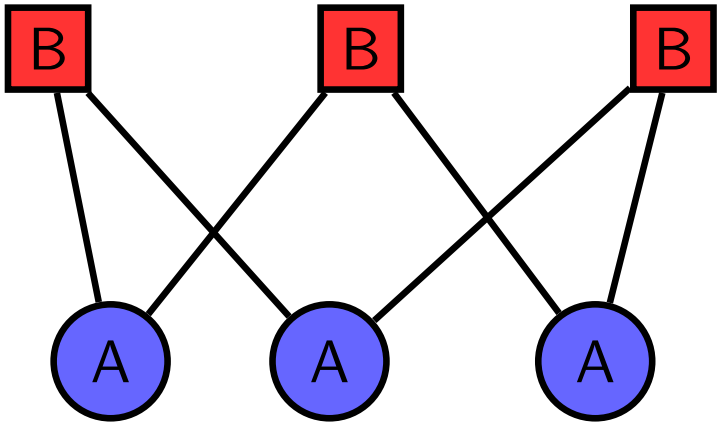

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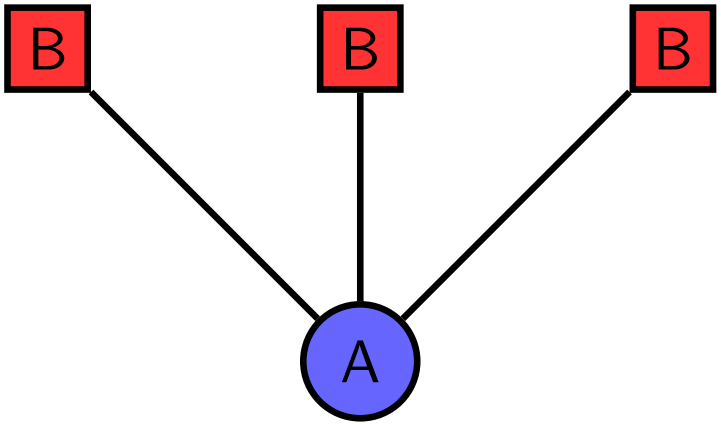
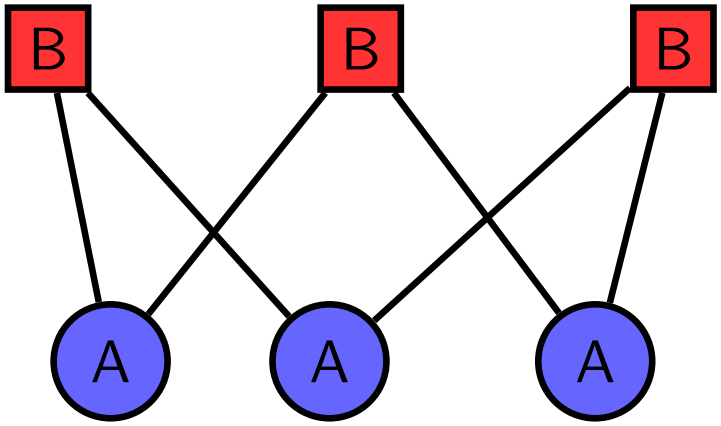
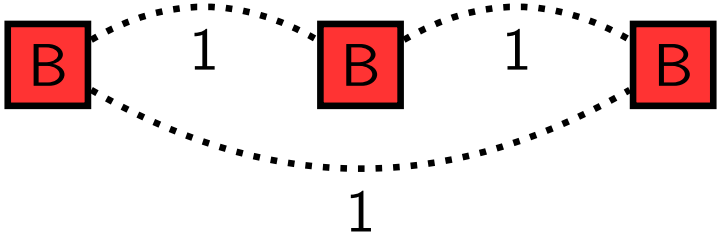




are both projected in



We-Sport: a sparse network

We consider a snapshot of the entire network:

- 1680 athletes

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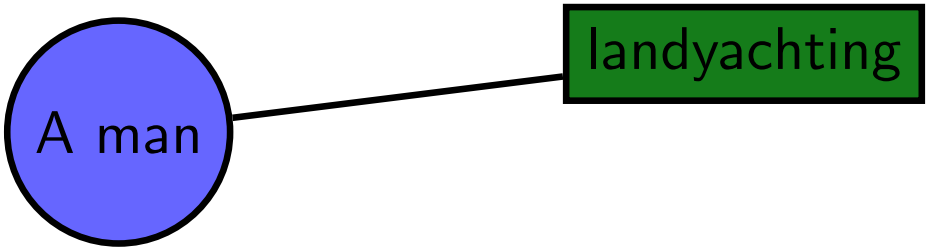
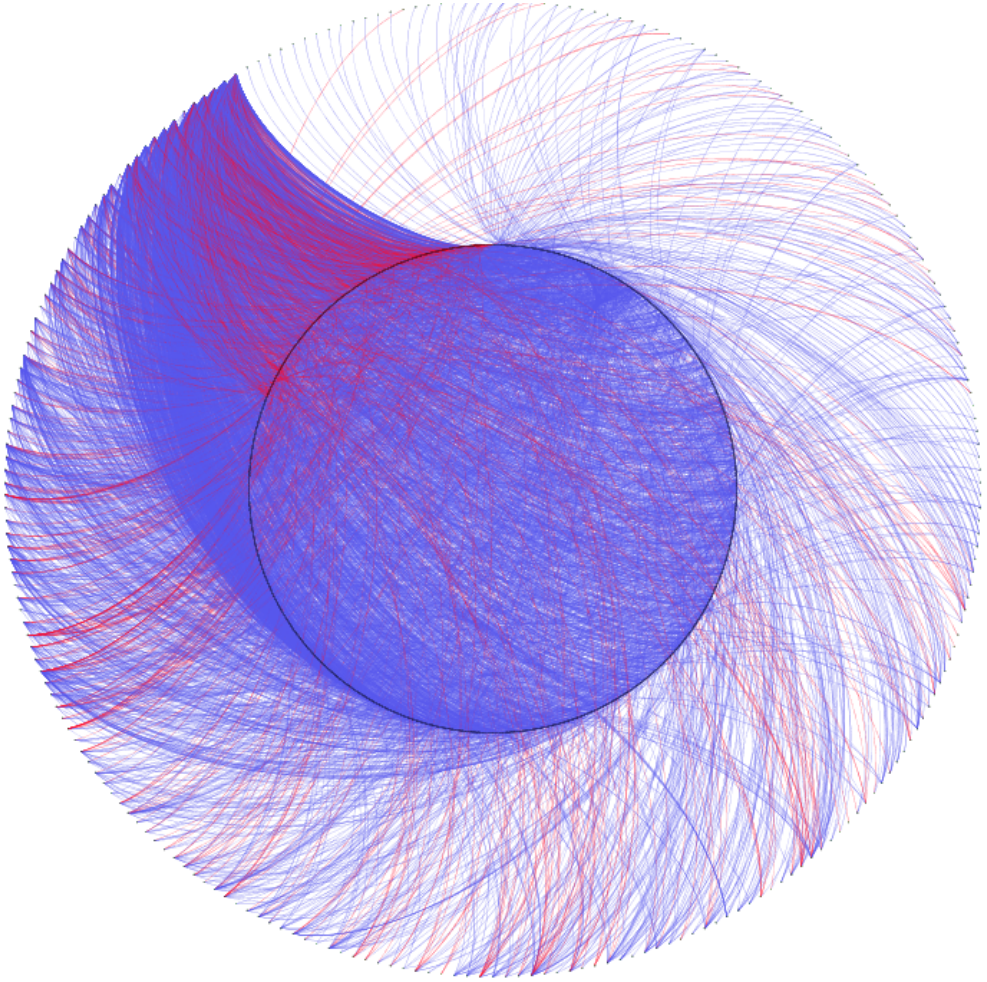


Graphical representation of bipartite We-Sport network



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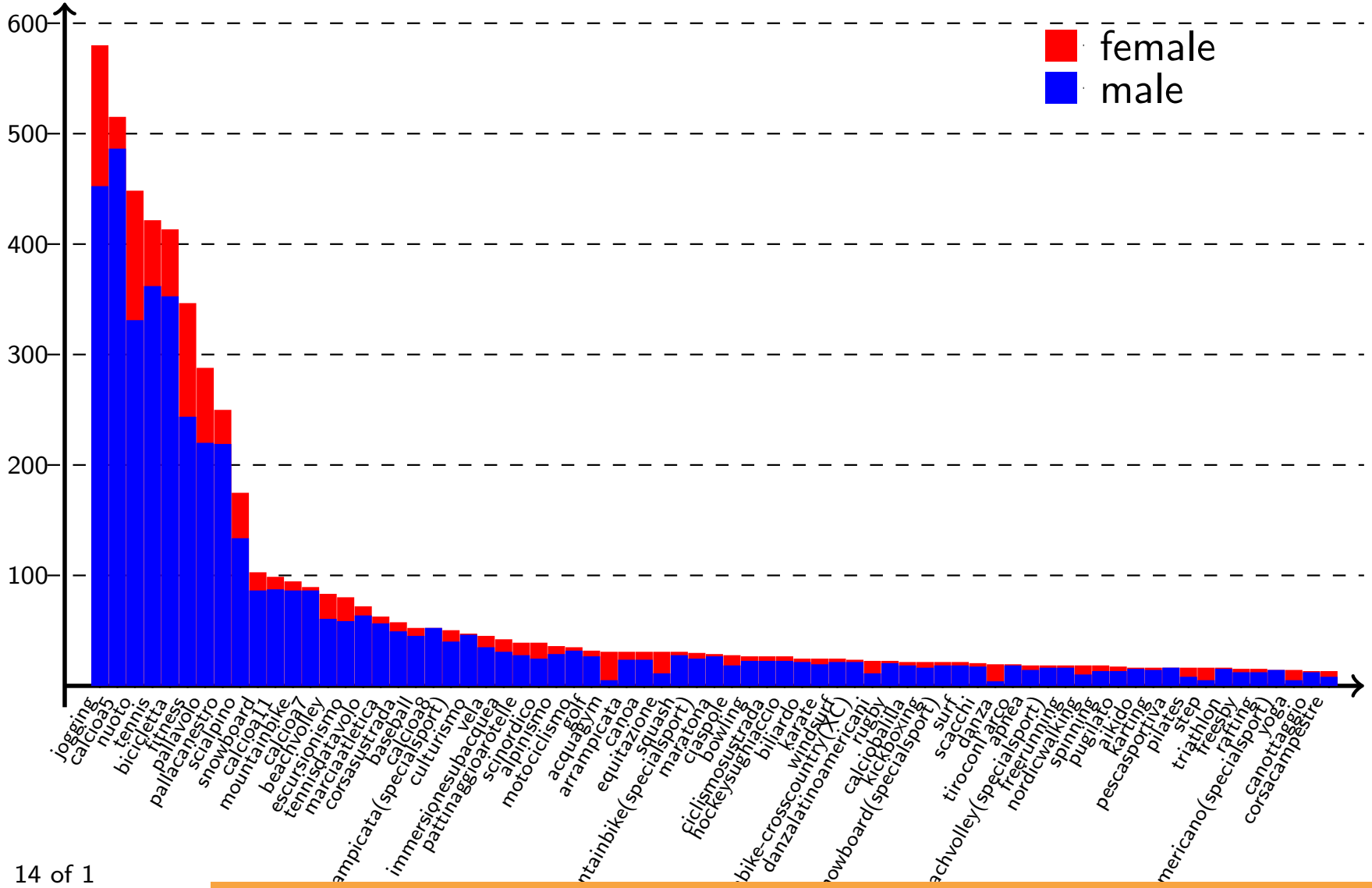
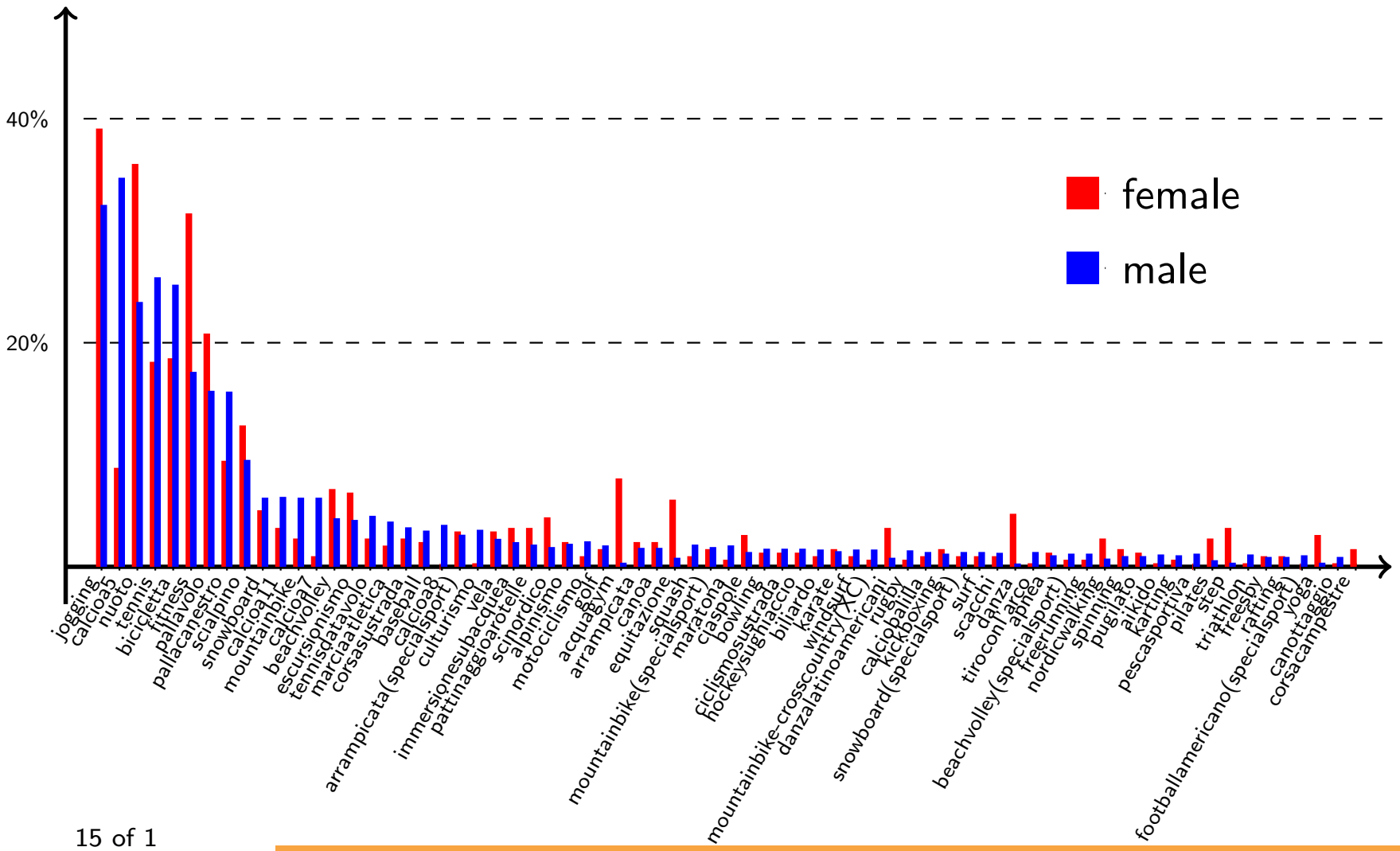
The 70 most played Sports



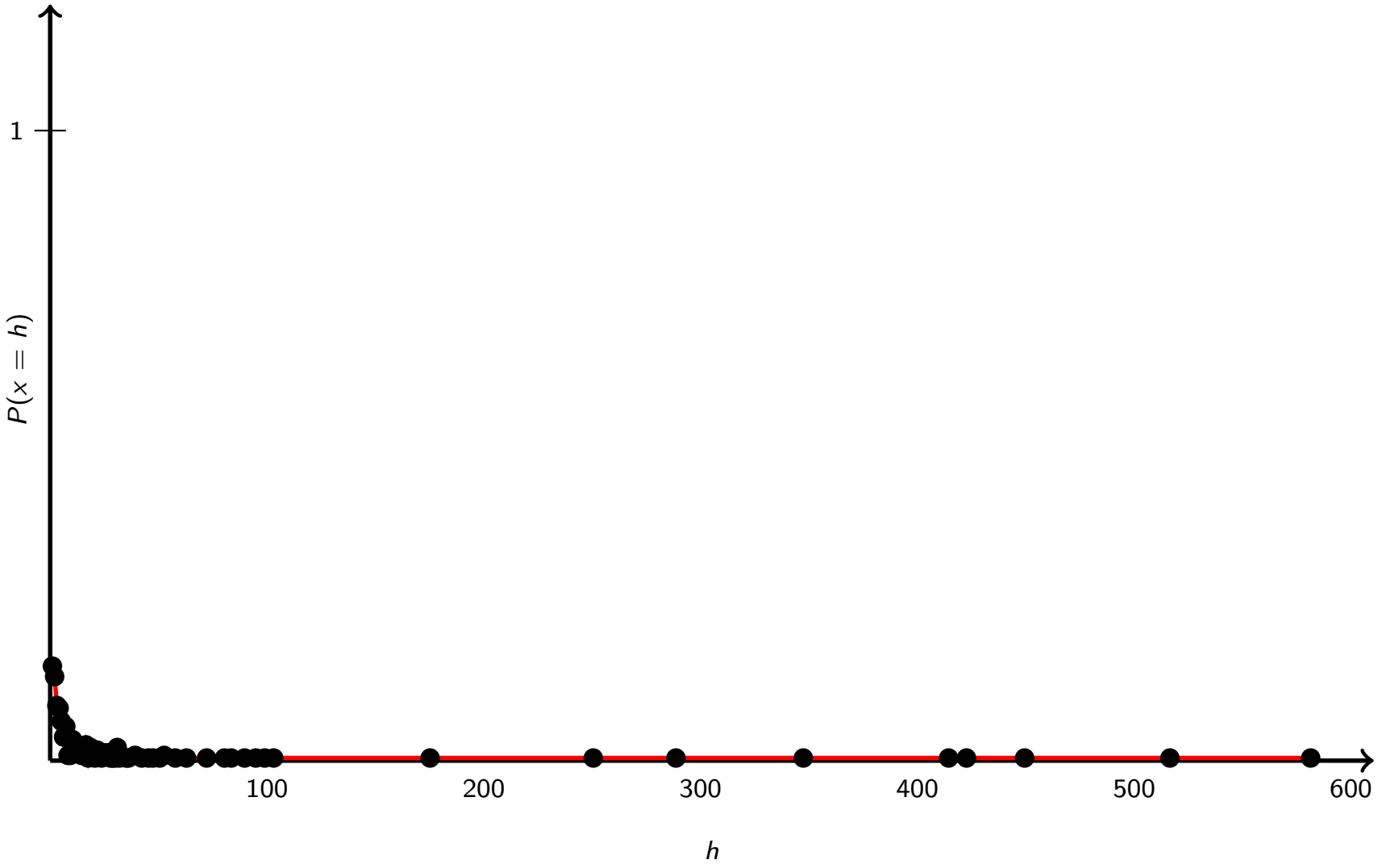
The 70 most played Sports: gender frequencies



A complex network

partition	mode	median	$\langle k \rangle$	$\langle k^2 \rangle$	$\frac{\langle k \rangle}{\langle k^2 \rangle}$
sport	1	5	25.54	$6.183 \cdot 10^3$	0.0041
athletes	1	3	3.63	23.78	0.1530

Degree distribution: sport nodes



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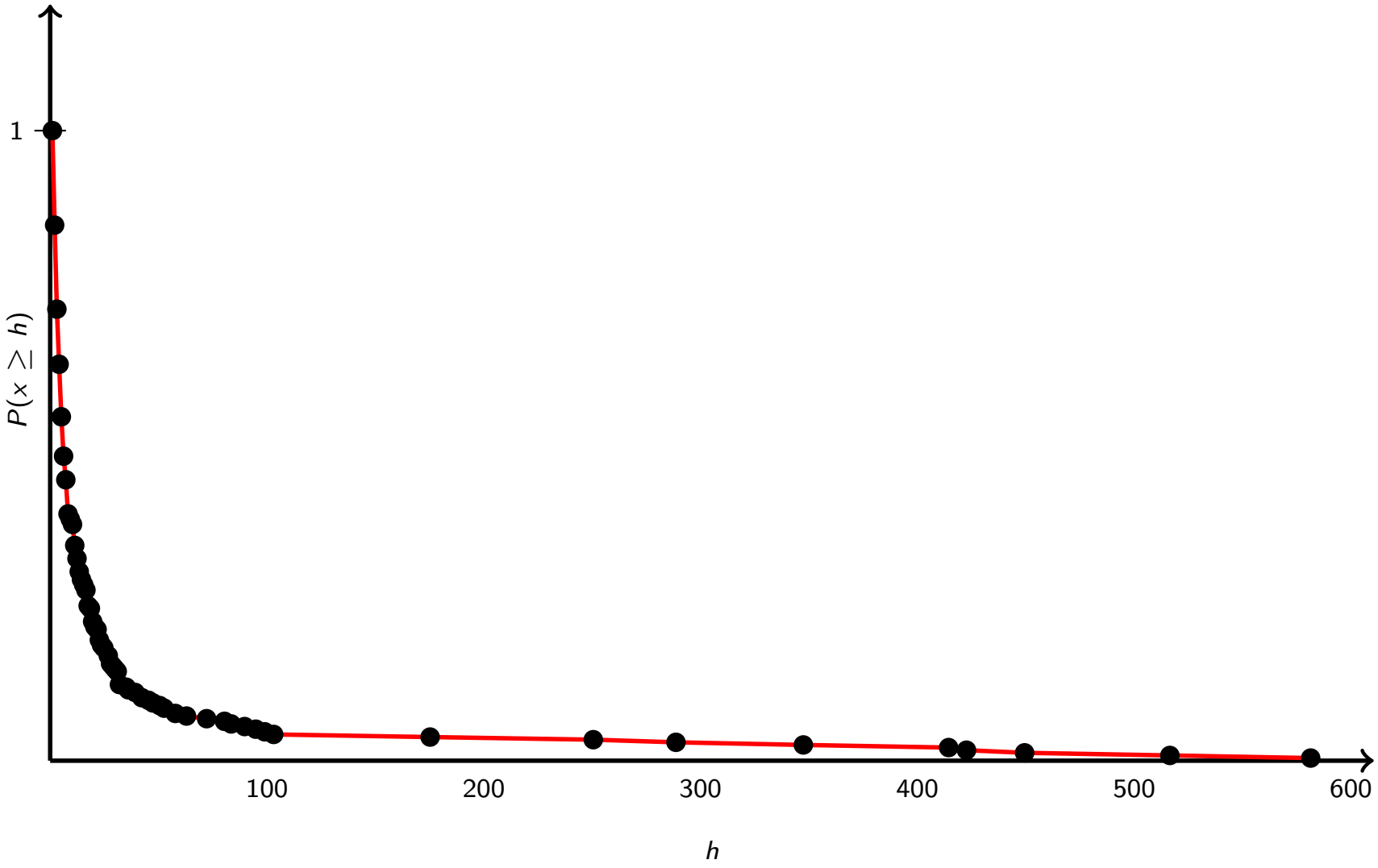
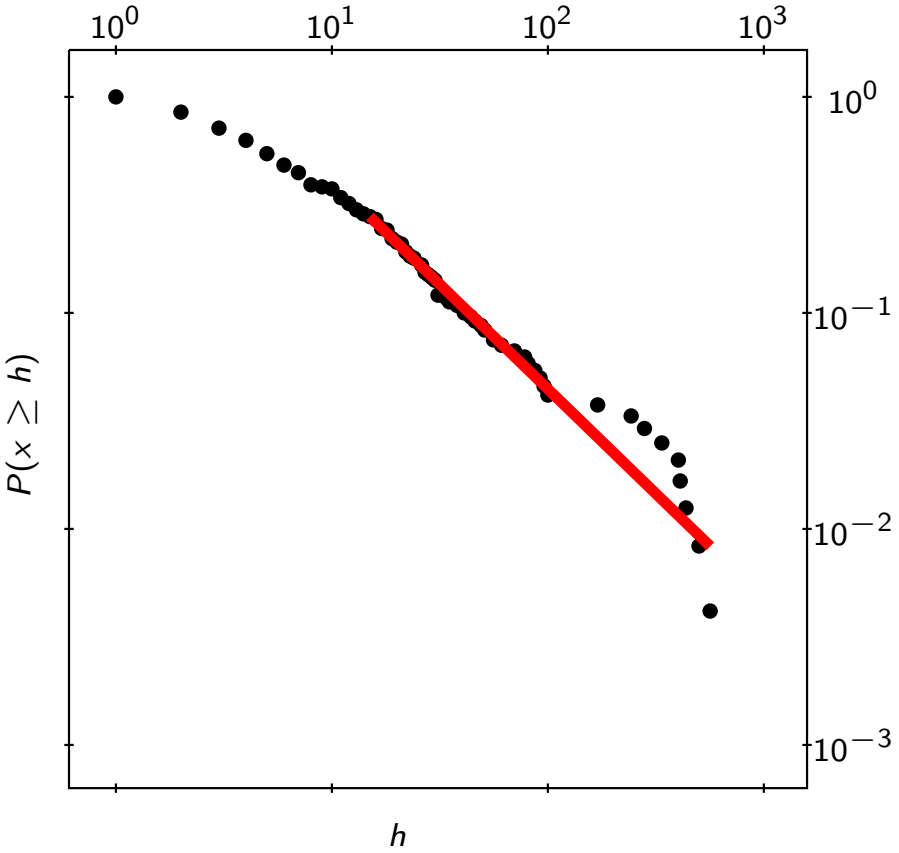


Degree distribution: sport nodes logarithmic scale



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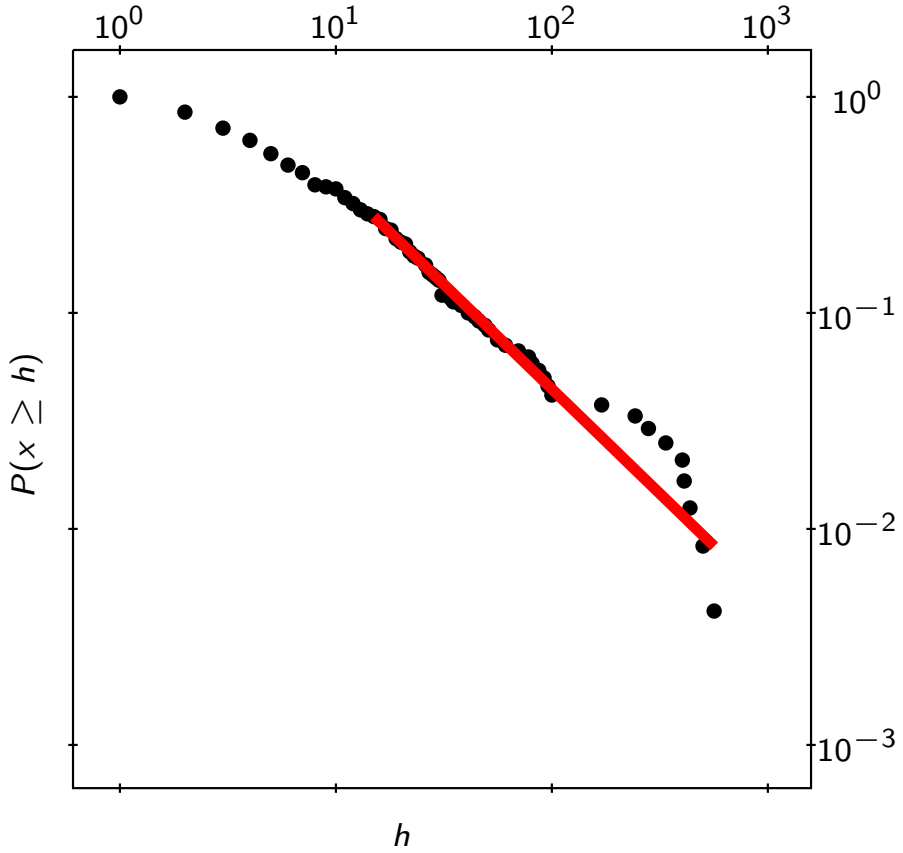

$\alpha = 1.96$

$x_{\text{min}} = 15$

p-value$=$ 0.5670

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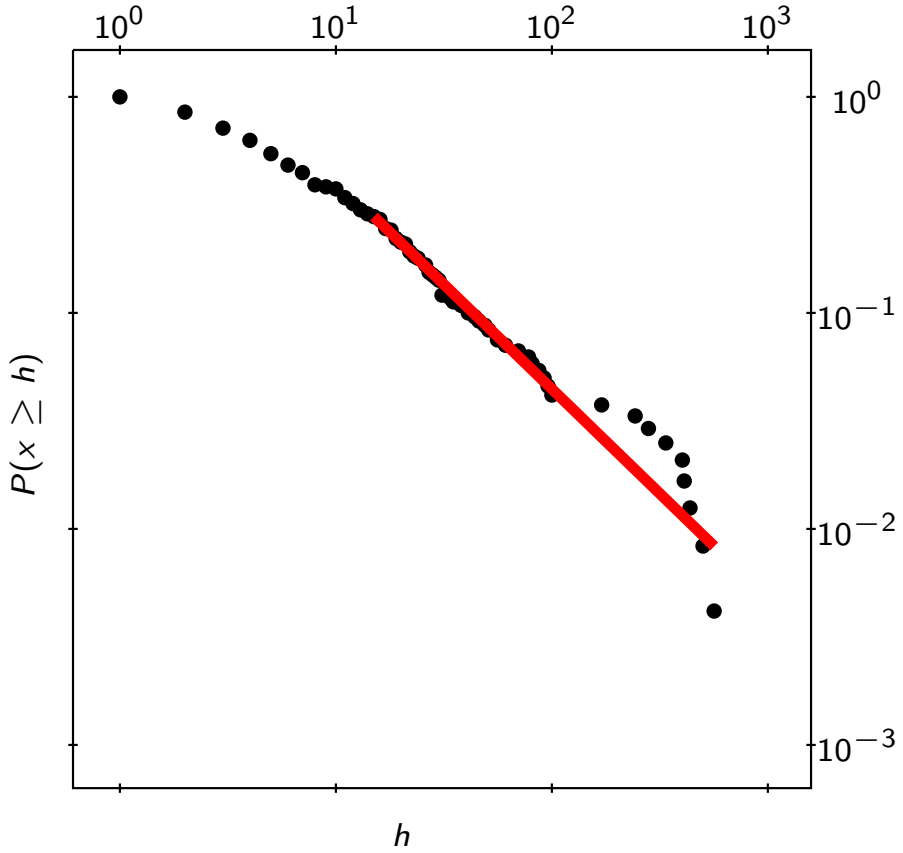

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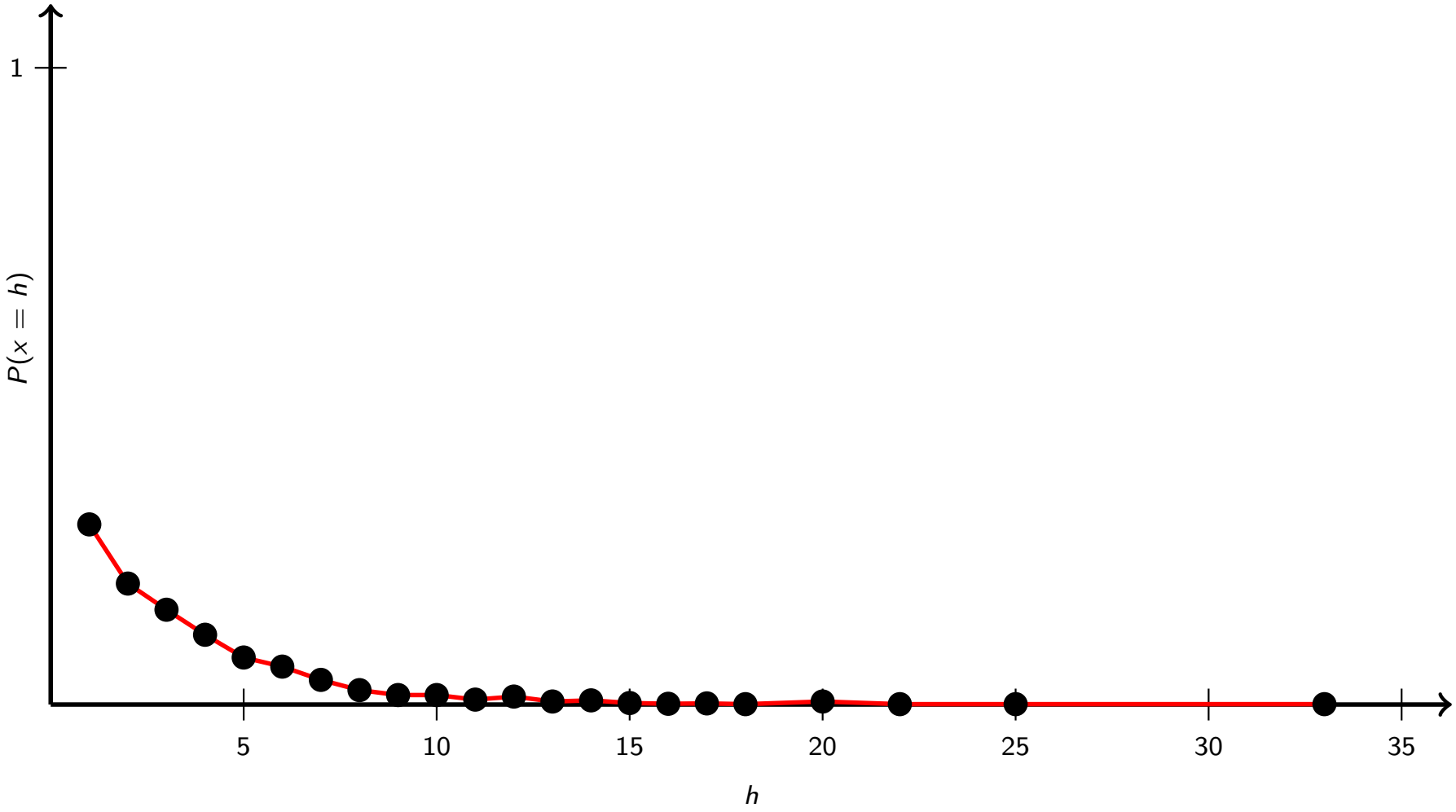
	bin.neg.		Poisson		exponential		Weibull		log-normal		Yule		power law + cutoff	
	LR	p	LR	p	LR	p	LR	p	LR	p	LR	p	LR	p
sport - nodes	4.14	**0.00**	4.09	**0.00**	4.31	**0.00**	0.13	0.89	−0.35	0.72	−0.004	0.94	−0.53	0.30

Degree distribution: athletes nodes



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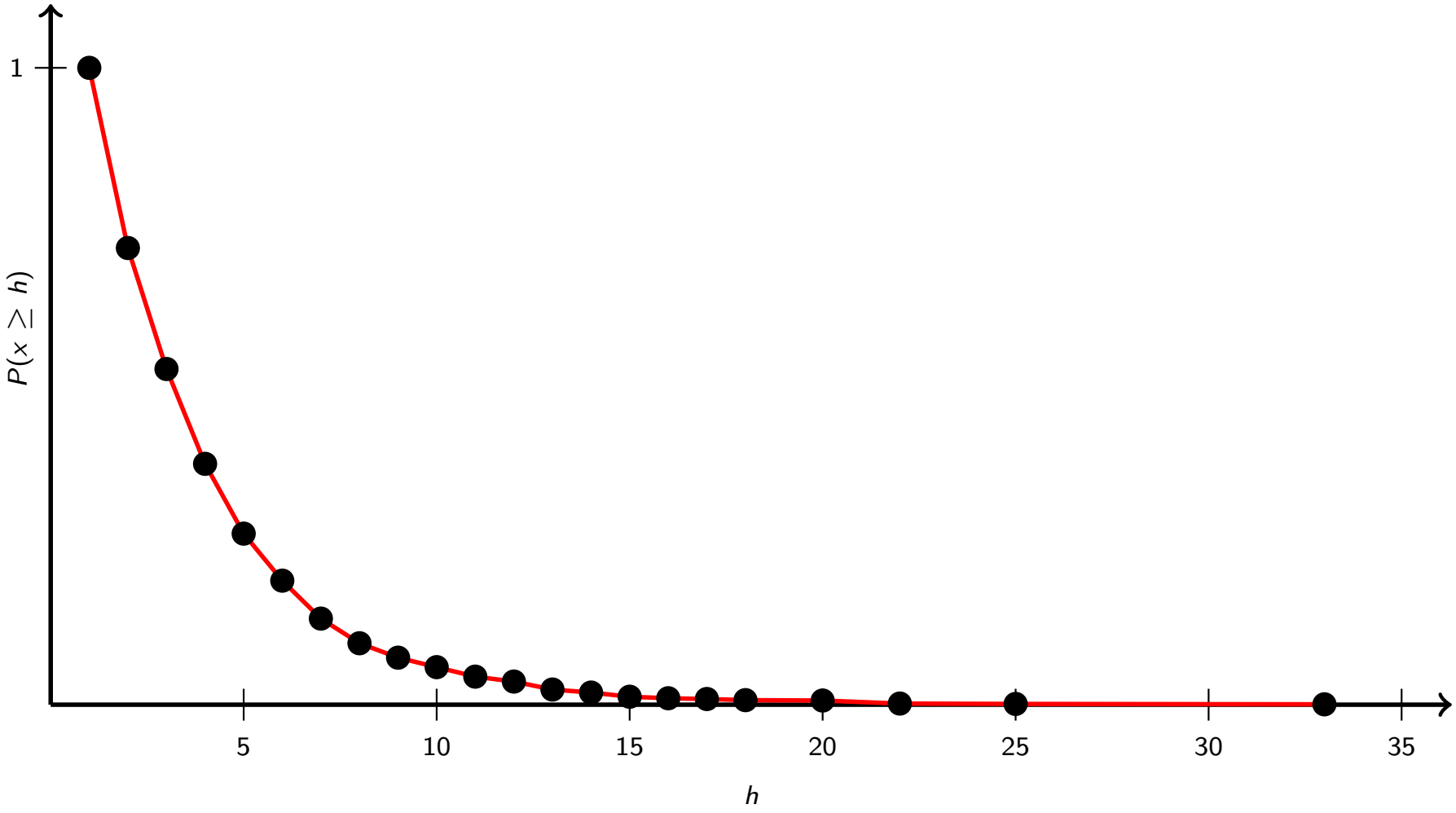
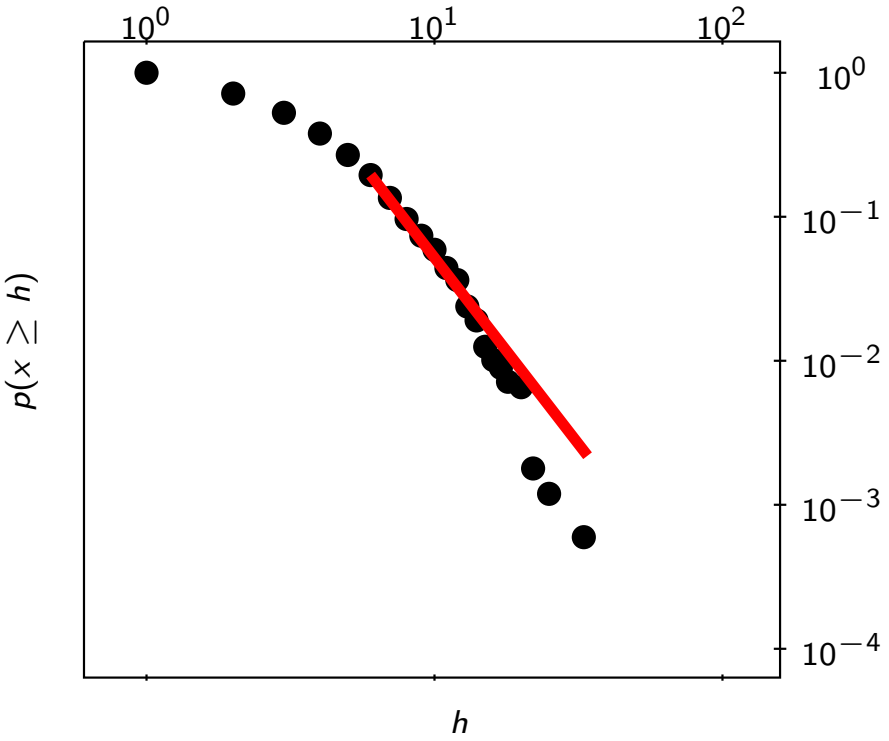


Degree distribution: athletes nodes logarithmic scale



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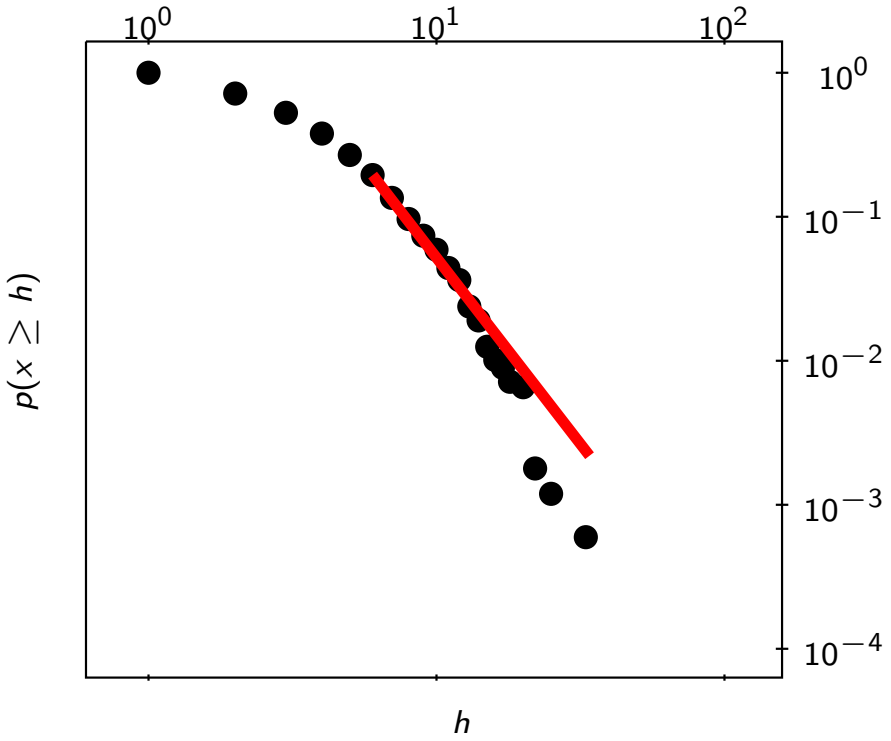

$\alpha = 3.49$

$x_{\text{min}} = 6$

p-value$= 0.0730$

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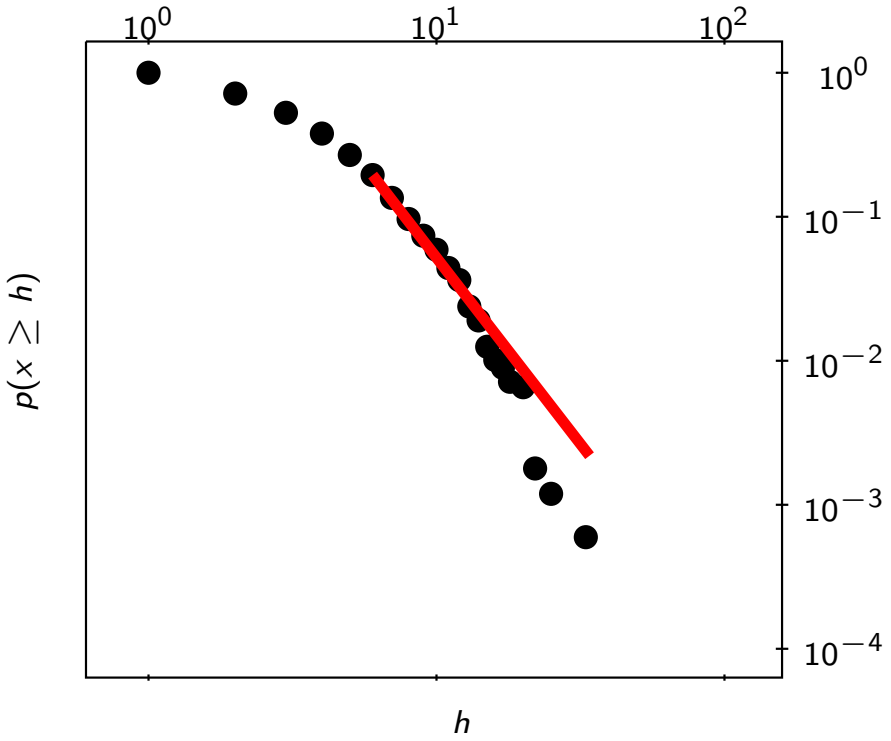

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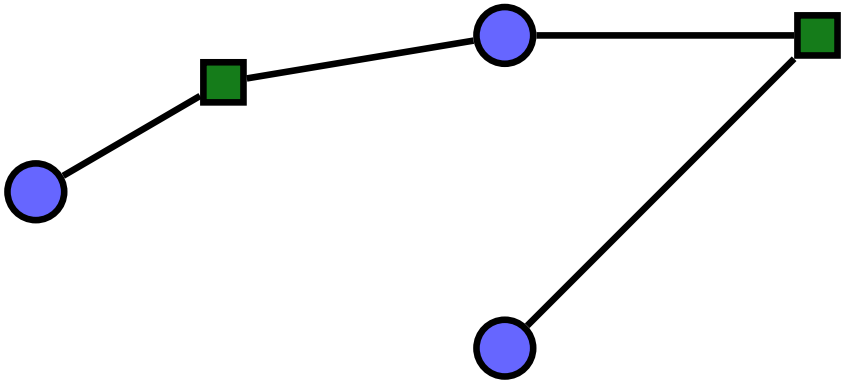
	LR	p	LR	p	LR	p	LR	p	LR	p	LR	p	LR	p
athletes - nodes	3.73	**0.00**	3.09	**0.00**	−0.83	0.40	−2.16	**0.03**	−2.12	**0.03**	−3.94	**0.00**	−6.62	**0.00**

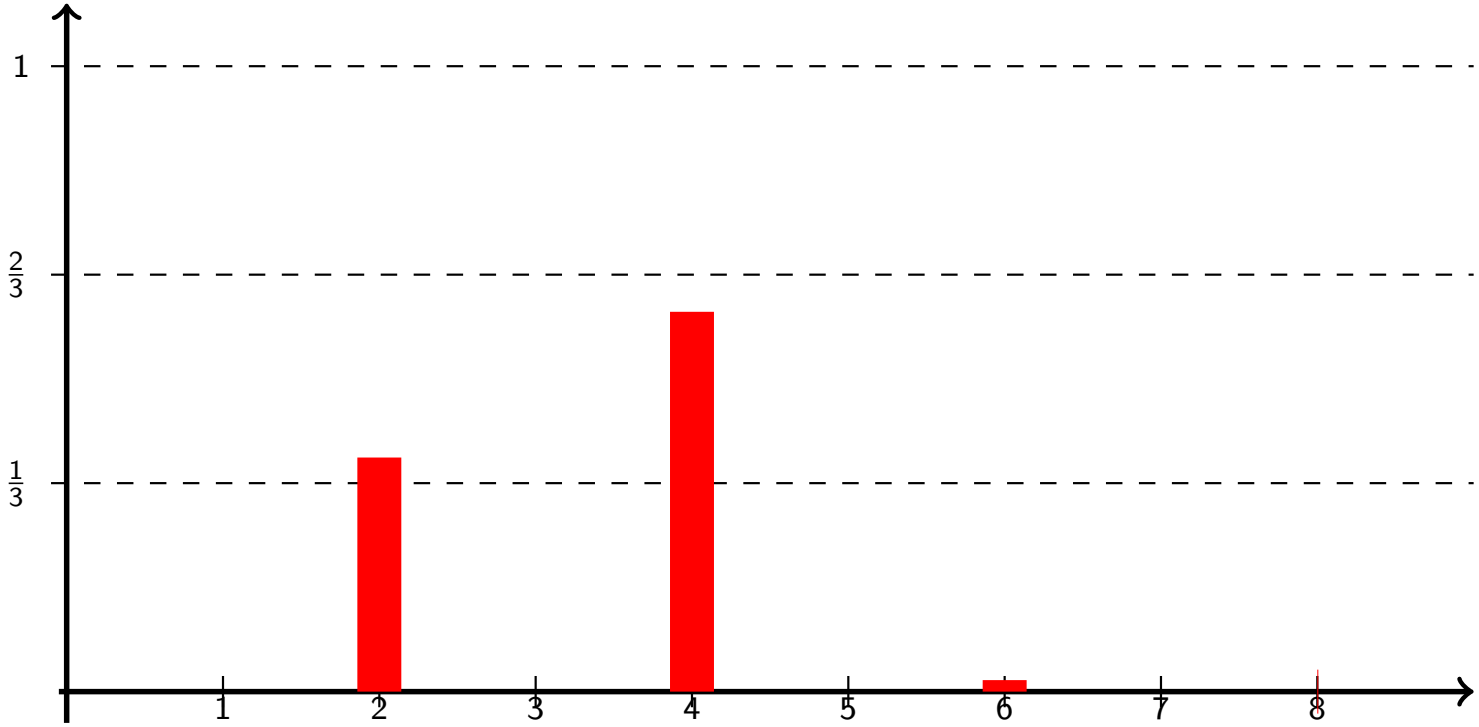
Nodes distance



The maximum distance between every pairs of nodes in a graph is defined as the diameter of the graph. We observe a diameter of 8 but on average the shortest path between nodes is 3.33.
We are in presence of so called small-world phenomena.

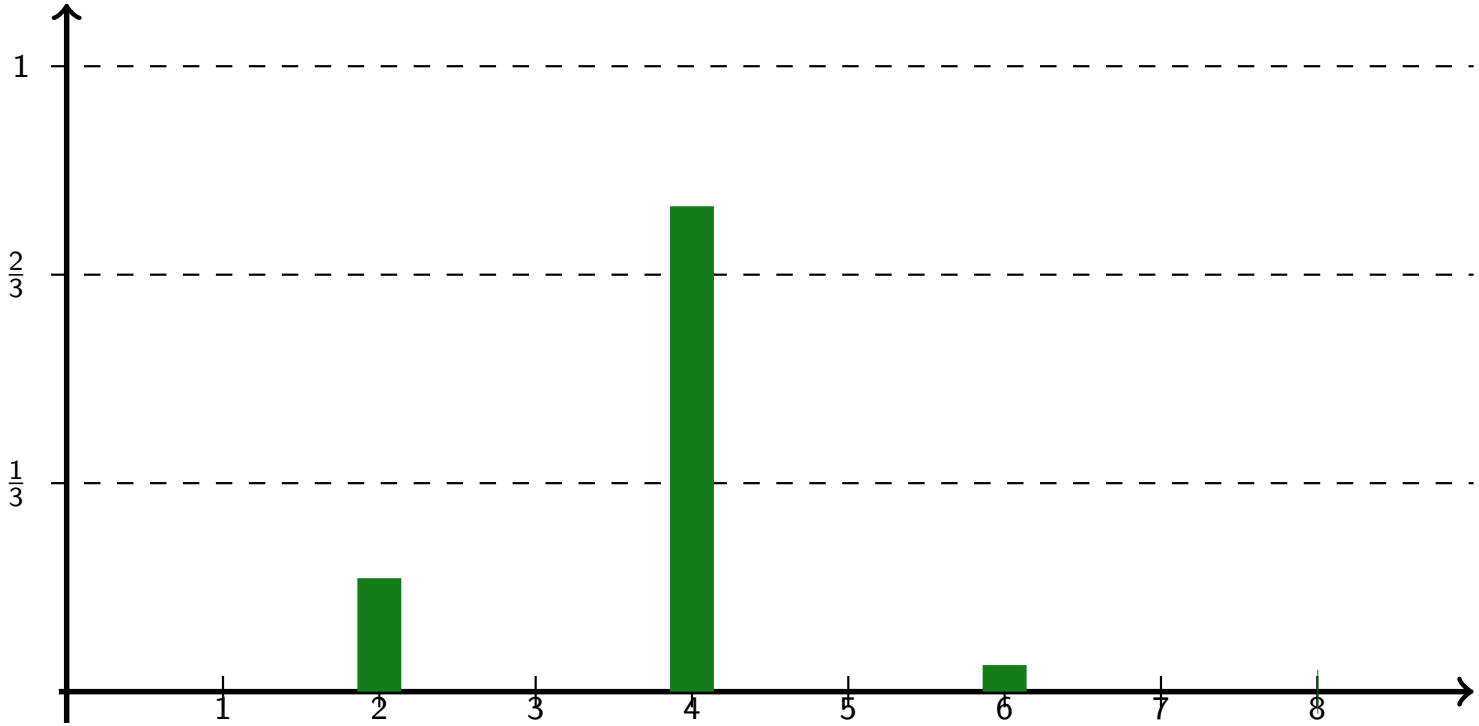
The distance distribution

the athletes-athletes distance



The distance distribution

the sport-sport distance



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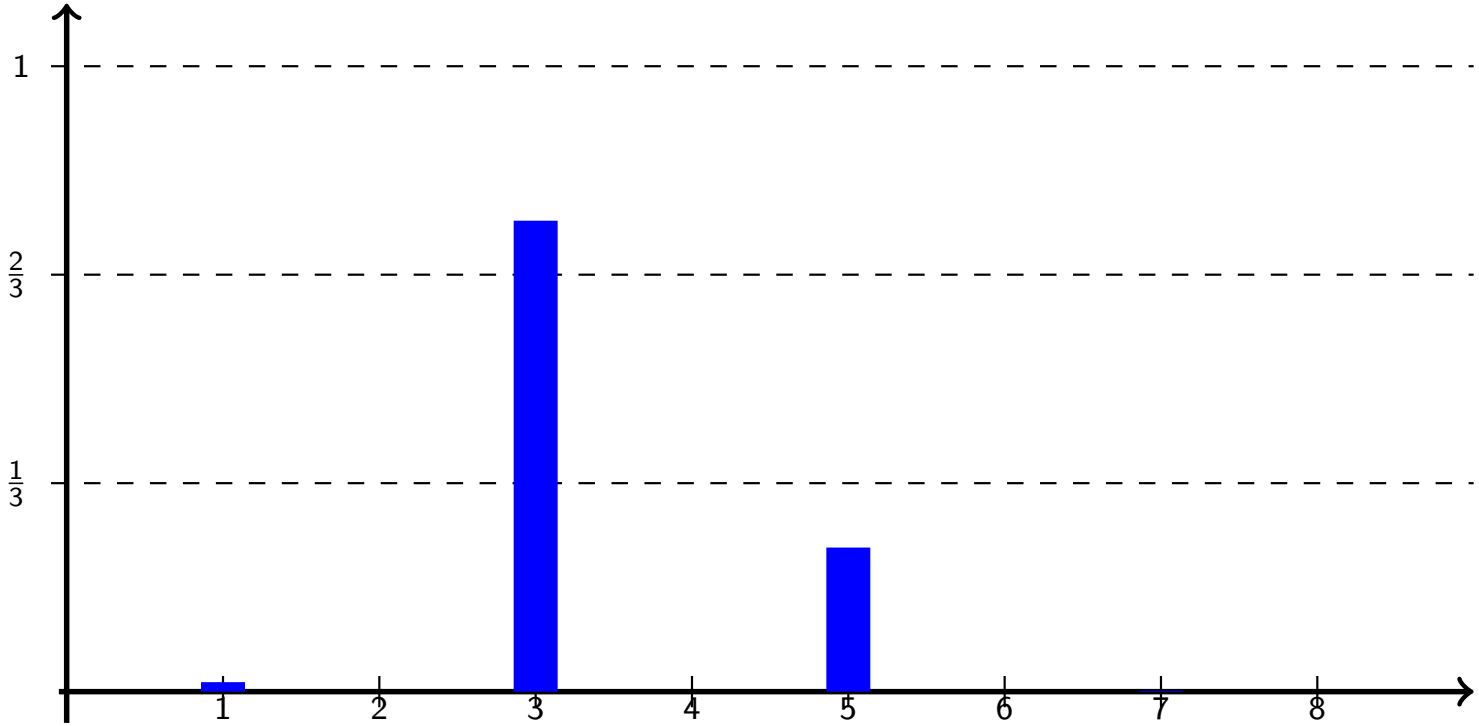
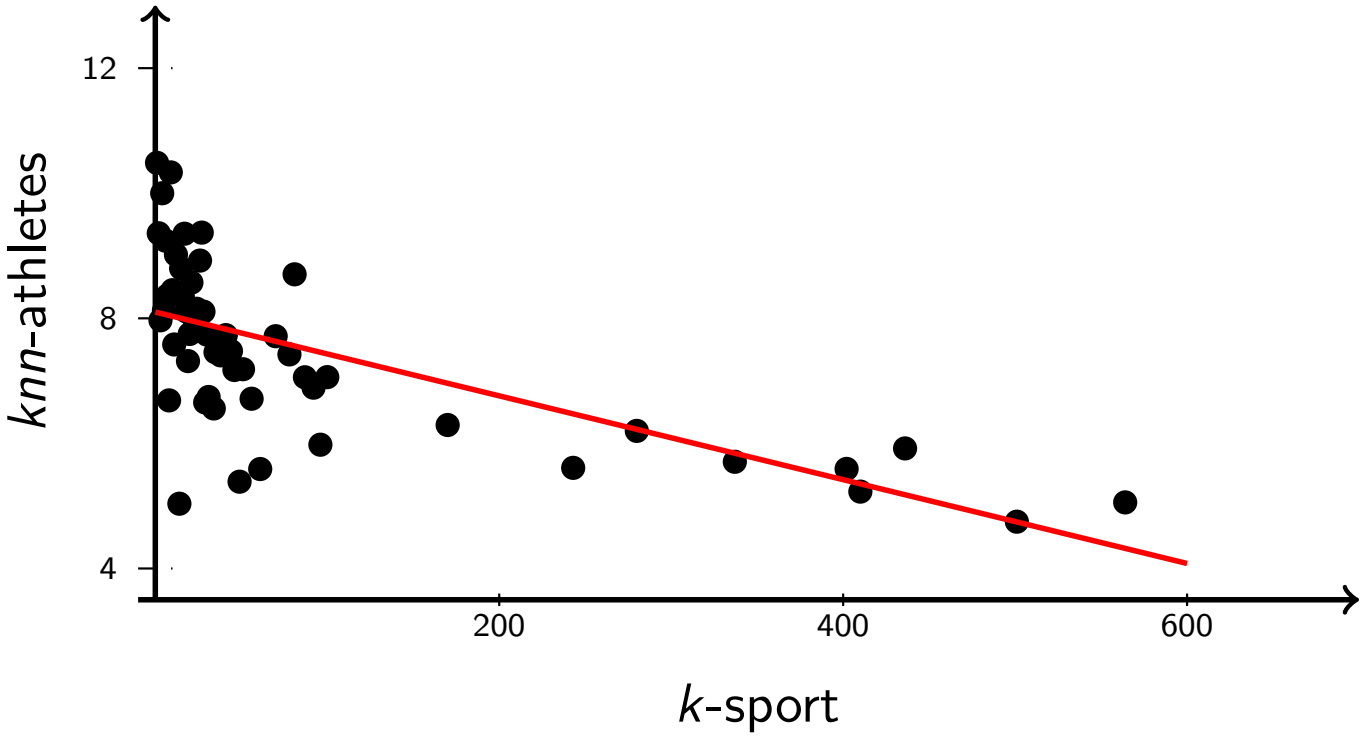


Assortativity in bipartite

We-sport network shows a disassortative behaviour: the Pearson coefficient is -0.2425.
Moreover if we calculate the nearest neighbor degree:



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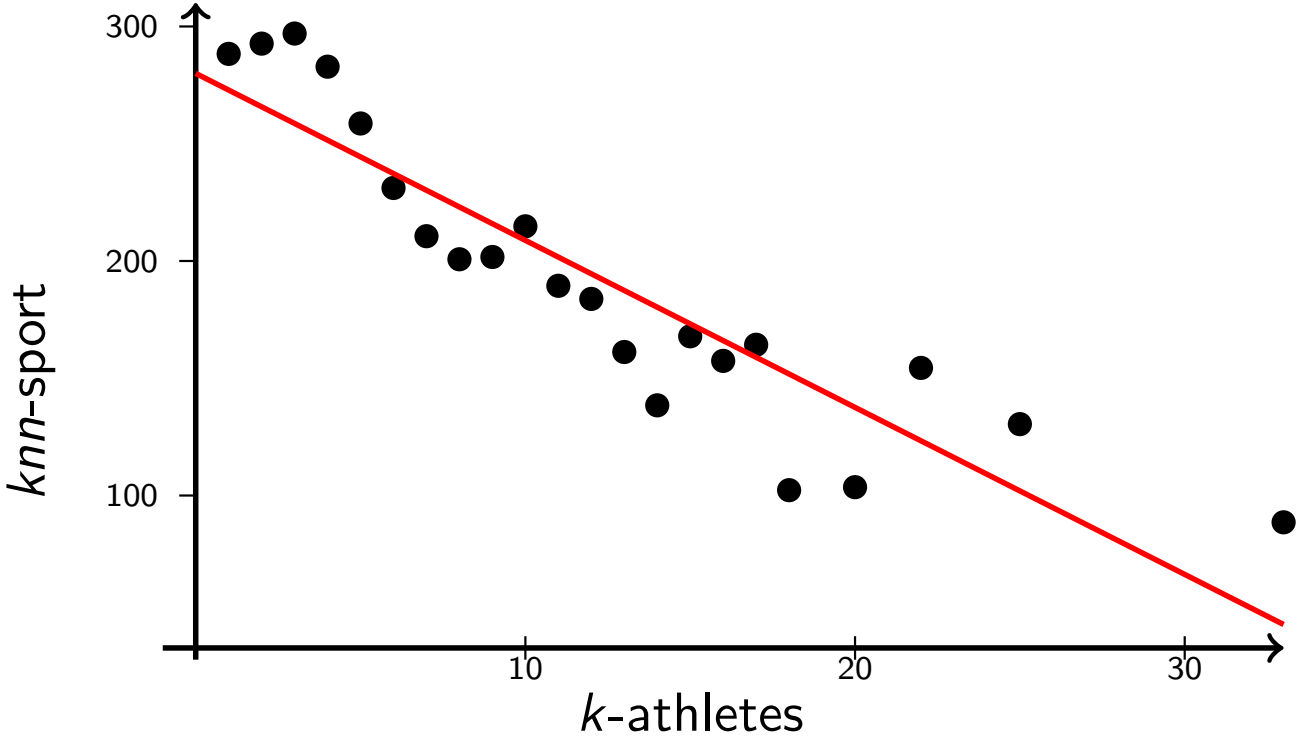
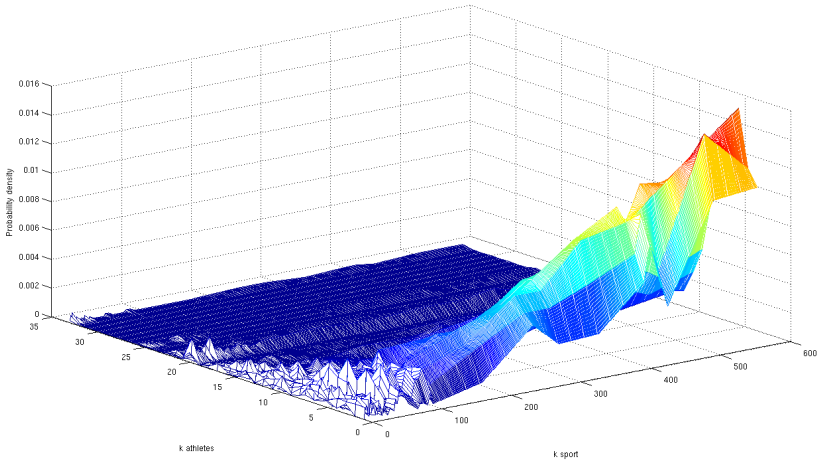
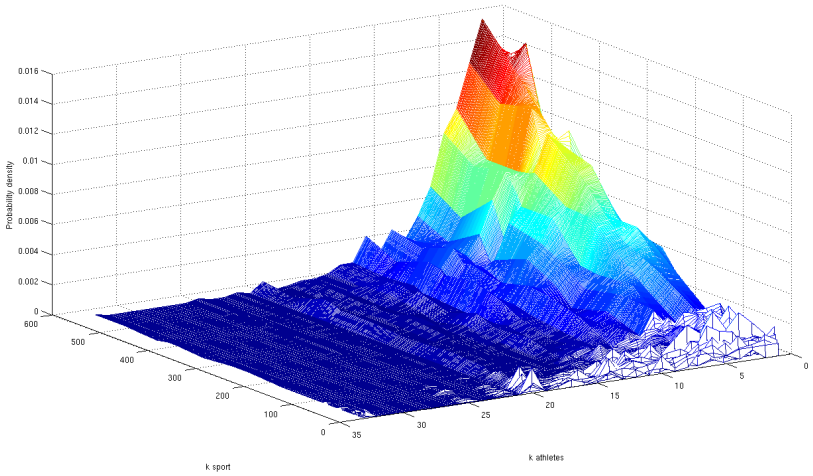
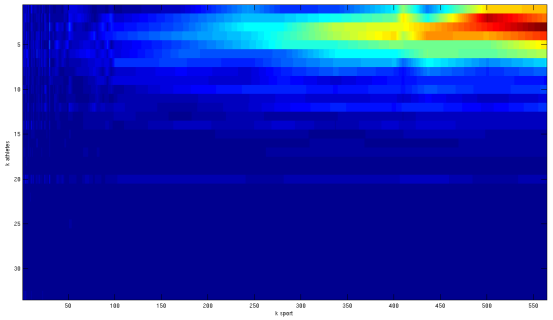


The joint probability







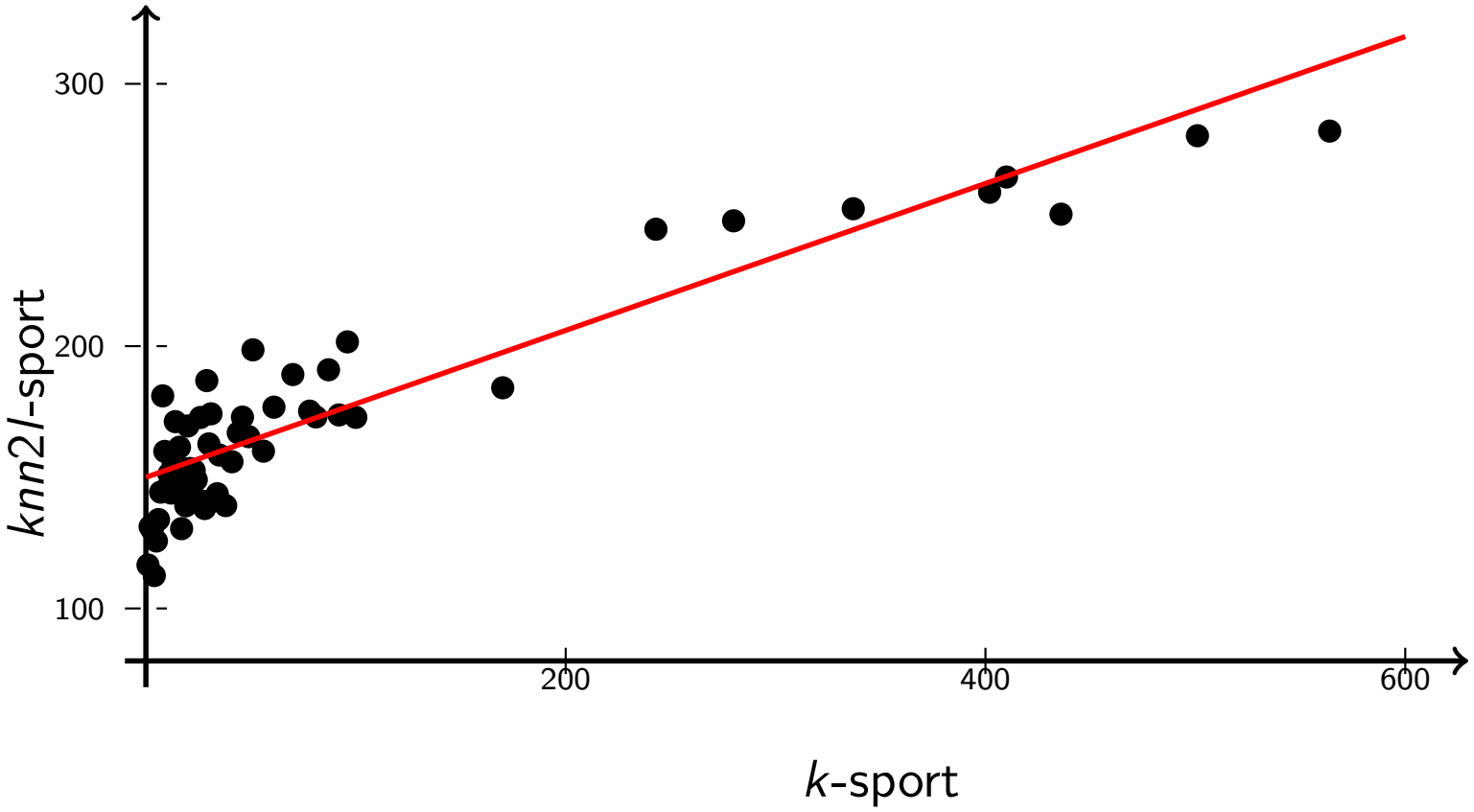
2-length assortativity in bipartite

We want to try to answer the question:

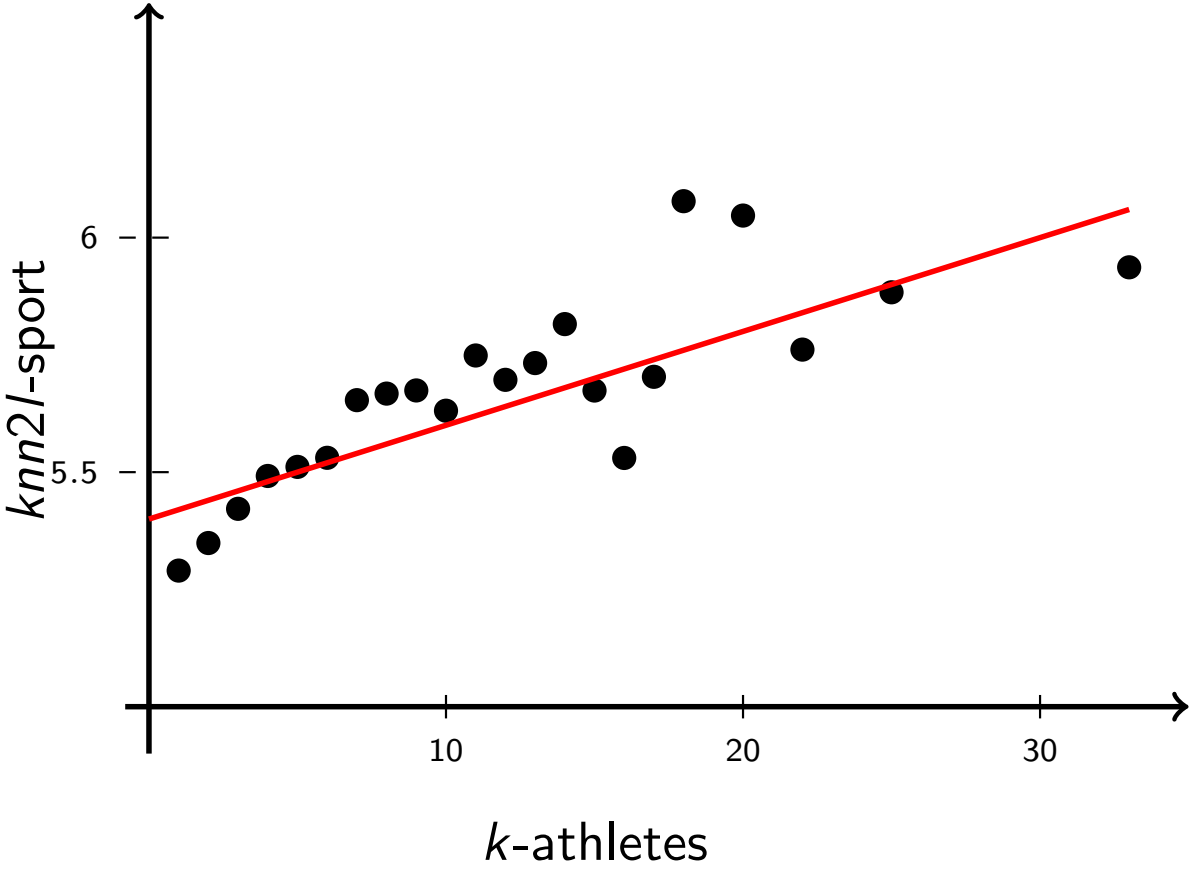
> *do people choose sports that connect them with similar people or not?*

Therefore we analyze the 2-length assortativity: we observe a weak assortative behavior for athletes-nodes (0.0326) and stronger for sport-nodes (0.2620)

2-length assortativity in bipartite



2-length assortativity in bipartite



The Clustering Coefficient for Biparite Networks

Again in order to understand the aggregation behavior of athletes we try to understand if people prefer to connect with other sharing the same sport's preference. Hence we define a similarity matrix cc which counts for each couple of athletes the number of sports they share:

$$|N(v) \cap N(u)|$$

then we can normalize that matrix. Le Blond et al., Latapy et al., and Borgatti suggest the three following denominator:

- $\min(|N(v)|, |N(u)|)$ for $cc_{\underline{\bullet}}(u, v)$

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The clustering coefficient II

From the similarity matrix we can calculate the clustering coefficient of each nodes.

$$cc(v) = \frac{\sum_{u \in N(N(v))} cc(v,u)}{|N(N(v))|}$$

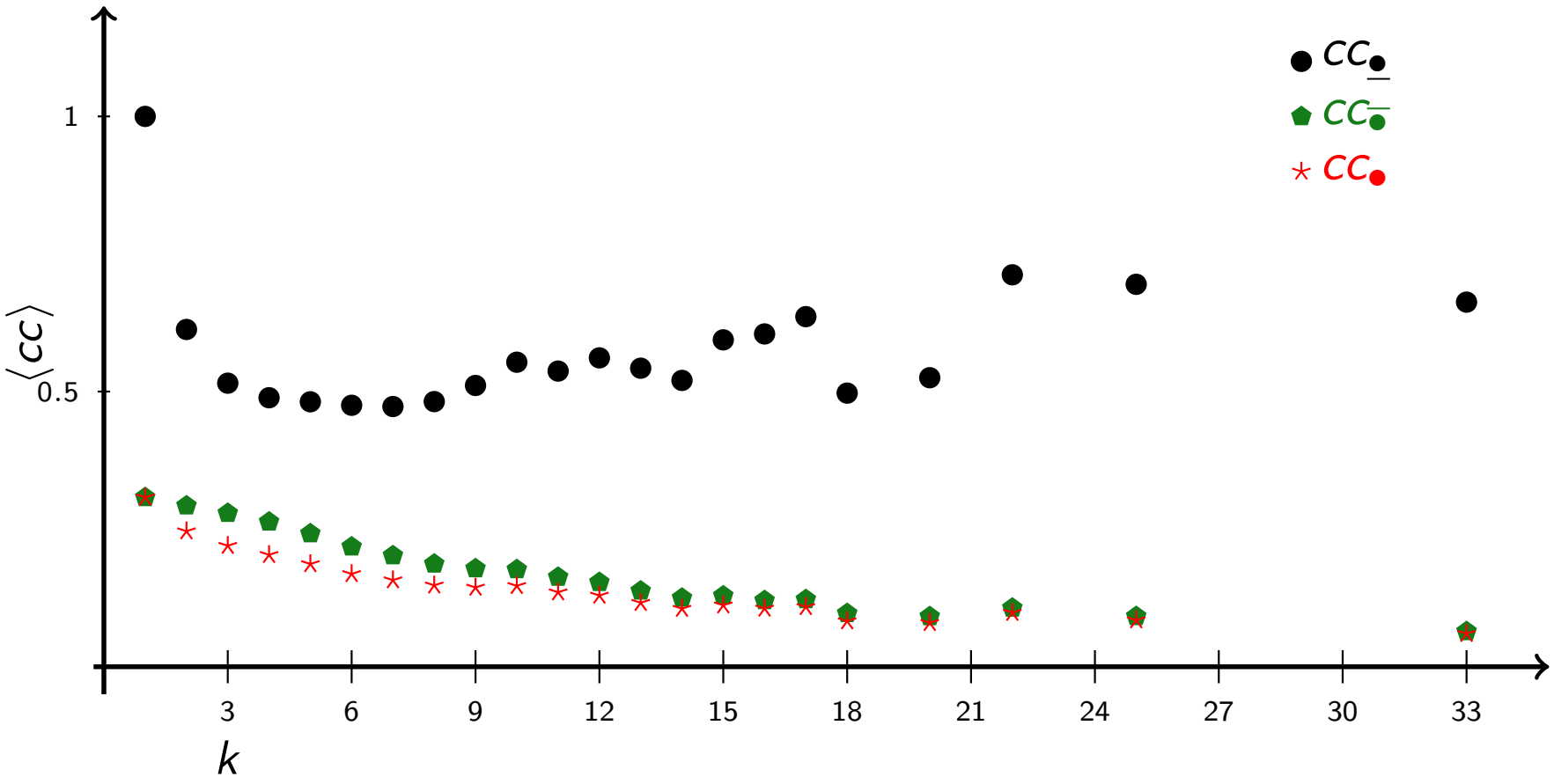
and from that the clustering coefficient of A-partition:

$$cc = \frac{1}{|A|} \sum_{v \in A} cc(v)$$

graph	$cc_{\underline{\bullet}}$	$cc_{\overline{\bullet}}$	$cc_{\bullet}$
athletes	0.6628	0.2672	0.2315
sport	0.4126	0.0615	0.0536

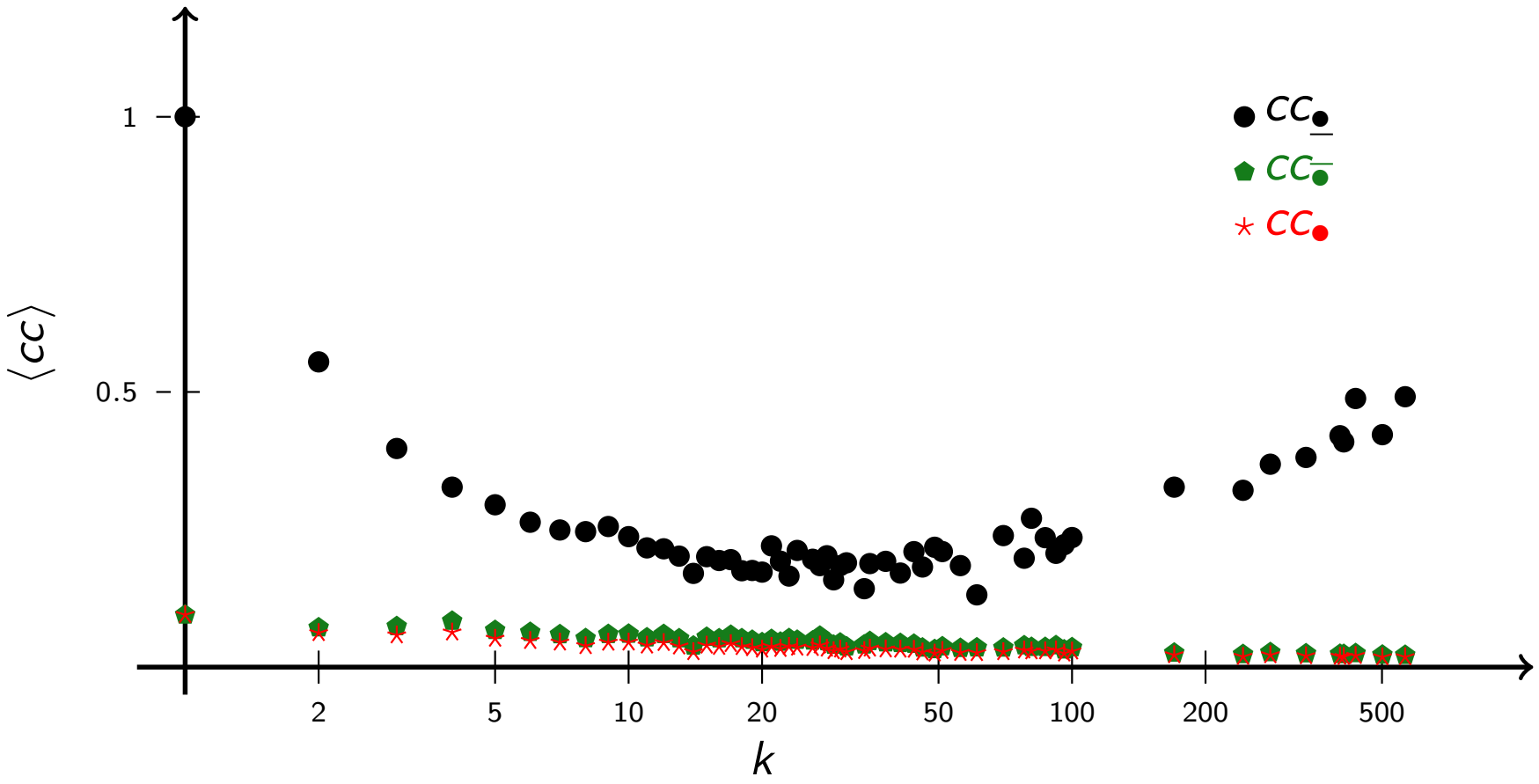
The clustering coefficient II

The athletes case

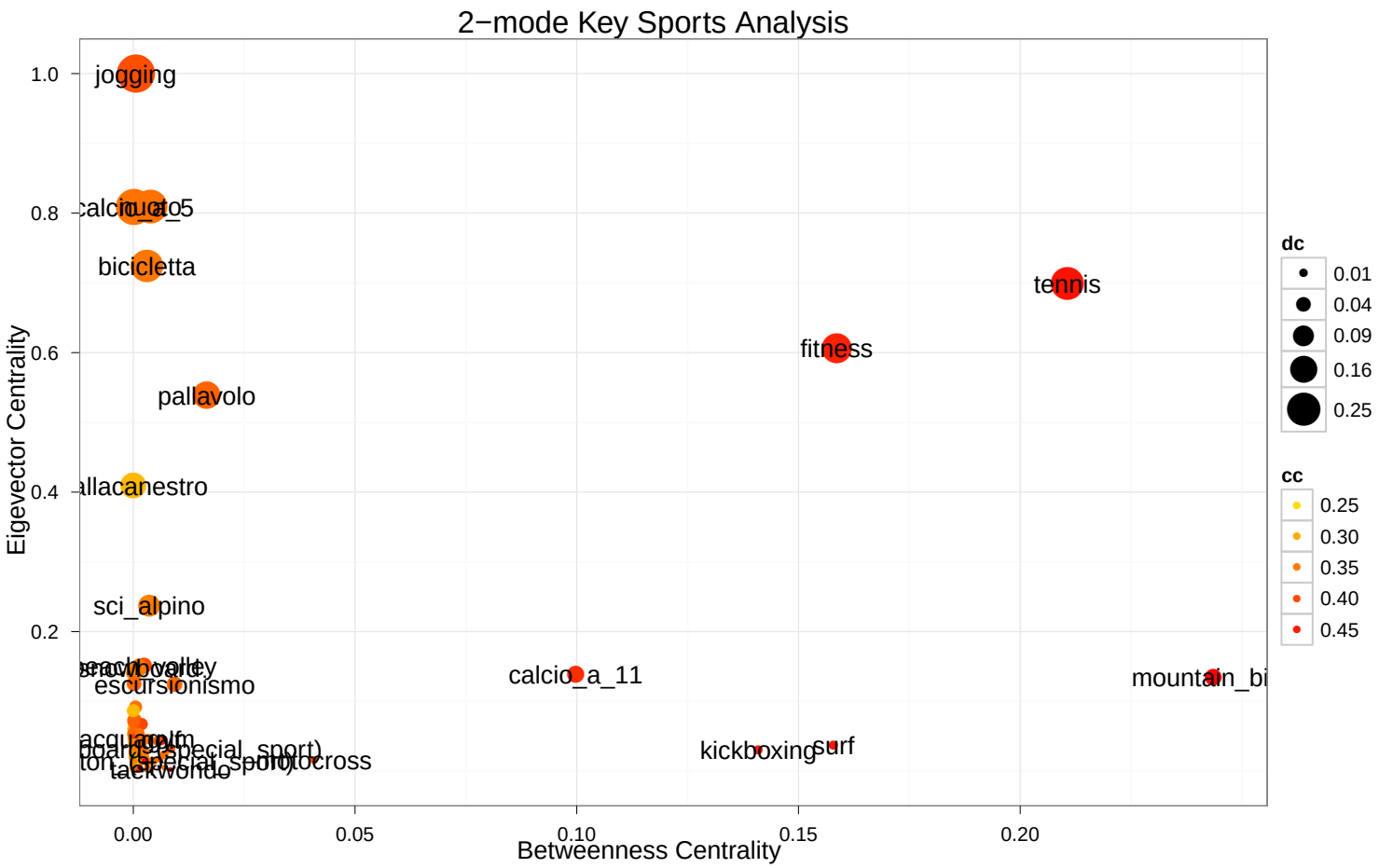


The clustering coefficient II

The sport case



The centrality



An application: which is the best sport to meet girls?

Contacts

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