Intrinsic Properties of Epidemiological Dynamics on Bipartite Networks

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Bipartite Networks

In bipartite networks nodes are divided in two families and edges are allowed only between individuals belonging to different families.

Vector-Borne Diseases

A disease is called vector-borne when the pathogen is transmitted from an infected individual to another individual by a blood-sucking arthropod, called vector.

An example: Lyme-Borreliosis

Lyme Borreliosis (LB) is the most common vector-borne disease of humans in temperate climates. It can affect at different threat levels several tissues in the human body: skin, heart, nervous system, eyes, kidneys, and liver.

LB's pathogenic agent is spirochetal bacteria belonging to the Borrelia genus. Vectors are hard ticks of the genus Ixodes (Europe I. ricinus), while hosts are small-medium vertebrate such as reptiles, birds and mammals depending on the host-seeking tick stage.

Infections Spreading on Bipartite Networks

The transmission of pathogens over a bipartite networks was firstly analyzed by Gómez-Gardeñes and colleagues for heterosexually transmitted diseases.

Those results were generalized by Bisanzio and coauthors and applied to vector-borne diseases. In that paper an important result was proven: if until then the aggregative behavior of vectors on unipartite networks structure sharing the same topological proprieties; pathogens become endemic for lower probability transmission than those need to spread over unipartite networks structure sharing the same topological proprieties; pathogens show an oscillating behavior which has an attractive biological interpretation.

Results presented here are of some interests from a biological point of view.

- The fact that there exist scale free random bipartite networks on which the transmission probability needed for a pathogen to became endemic tends to zero, in the limit of large population size, while this does not happens on same but unipartite networks could mean that pathogen could take advantage of the peculiar transmission route described by bipartite networks.

- Moreover the oscillating behavior observed in infection spreading over synthetic bipartite networks is observed in real world: e.g. tick nymphs are active in Spring while larvae are active during the Summer of the same year. Hence, in order for the pathogen to be passed to a next generation of larvae, hosts are required to keep the pathogen from Spring to Summer, having been first infected by nymphs and then transmitting to larvae.

Essential bibliography

- Bisanzio, Bertoldi, Tommaso, Amore, Ragagl, Mannelli, Giacobini, Provero - Modeling the Spread of Vector-Borne Diseases on Bipartite Networks - PLoS ONE, 5, 10, 2010
- Ferreri L, Venturini E, Giacobini M - Do Disease Spreading on Bipartite Networks Have Some Evolutionary Advantages? - LNCS 6823 - 2011

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Evolutionary Advantage?

We compare the epidemic threshold derived on bipartite networks with those derived on same but unipartite networks. In fact we compare the asymptotic behavior for enlarging population having degree distributed with two power-laws of exponents $\alpha_A$ and $\alpha_B$. Analytically we demonstrate that there exist couples of exponents $(\alpha_A,\alpha_B)$ for which the epidemic threshold tends to zero for bipartite but not for unipartite networks.

Oscillation Behavior

We generate a number of bipartite random networks with different sizes and connection probabilities $p$. Over these we let a disease spreads observing the following rules:

- a sound individual at time $t$ connected to at least one infected nodes will become infected at time $t+1$
- an infectious node at time $t$ will be susceptible at time $t+1$, no matter if it is connect or not with infective individuals!

What we observe is that after few iterations is that the system reaches an oscillating equilibrium. In fact we found a 2 periodic solution and even more amazingly we have an (anti-)synchronizion phenomena.

Analytical results

Let's consider random bipartite networks, where $A$ and $B$ are the two families of nodes, and two nodes are connected with probability $p$. For a node belonging to the $A$ partition the degree distribution is nothing but the binomial:

$$p(x|A) = \binom{k}{x} p^x (1-p)^{k-x}$$

and analogous for nodes belonging to $B$ partition. Calling $x(t)$ the prevalence of infected $A$-nodes at time $t$, and respectively $y(t)$ the prevalence of infected $B$-nodes, and using eq. (1) we could detect the probability for a $A$-node to have $k$ neighbors of which $h$ infected is nothing but:

$$p(h|A) = \binom{k}{h} p^h (1-p)^{k-h}$$

However, a susceptible node get the infection if at least one neighbor is infectious. Therefore, we sum previous for $h = 1, \ldots, A$. Thus, at time $t+1$ the prevalence in family A is nothing but

$$x(t+1) = (1-x(t))(1-y(t))^{p=0.15}$$

and similarly for the prevalence in family B.

Now, in order to check the 2-periodic solutions, which mean oscillations of period 2, we impose that concurrently $x_{t+1} = x_t - x_{t-1}$, and $y_{t+1} = y_t - y_{t-1}$, we numerically find the solutions using the Newton's method. We observe they change as the value of $p$ changes.

Discussions

Results presented here are of some interests from a biological point of view.

- The oscillating behavior observed in infection spreading over synthetic bipartite networks is observed in real world: e.g. tick nymphs are active in Spring while larvae are active during the Summer of the same year. Hence, in order for the pathogen to be passed to a next generation of larvae, hosts are required to keep the pathogen from Spring to Summer, having been first infected by nymphs and then transmitting to larvae.

Infections Spreading on Bipartite Networks

The transmission of pathogens over a bipartite networks was firstly analyzed by Gómez-Gardeñes and colleagues for heterosexually transmitted diseases.

Those results were generalized by Bisanzio and coauthors and applied to vector-borne diseases. In that paper an important result was proven: if until then the aggregative behavior of vectors on hosts was usually described by negative-binomial distributions, they showed on different datasets that the best fitting was power-law with exponents around 2.5.

This implies an important consequence: the epidemic threshold vanishes in the limit of large population sizes. In other terms, the condition for an endemic behavior, observed in several countries around the world, seems to be well supported by field data and by the network models. Moreover, on dynamical processes occurring on bipartite networks we observe two exciting phenomena.

- oscillations of period 2, which mean oscillations of period 2, we impose that concurrently $x_{t+1} = x_t - x_{t-1},$ and $y_{t+1} = y_t - y_{t-1}$, we numerically find the solutions using the Newton's method.

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